Blockbusting: Real Estate Brokers and Neighborhoods in Racial Transition*

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Abstract

The paper presents a continuous-time dynamic model of a neighborhood in racial transition as triggered by a fee-motivated real estate broker. Racial transition occurs in an initially all-white neighborhood when the broker steers white sellers toward black buyers. Racial transition leads to a higher rate of property turnover in the neighborhood but also to lower prices, and this is the trade-off faced by a broker. We show that racial transition will occur only when (i) white households have moderate racial preferences, (ii) the value of housing for black households outside the neighborhood is low, (iii) the broker is moderately patient, (iv) the arrival rate of offers is moderate, and (v) the number of brokers is limited. Otherwise, the real estate broker steers white households toward white buyers.

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> Suppose the bungalow came into possession of a Negro? What would happen to the rest of the block? . . . "*Relax,*" said the bungalow owner. "*I'm selling this through a white real-estate man. I won't even talk to a Negro.*"
>
> —Norris Vitchek, "Confessions of a Blockbuster" (July 14, 1962)

1 Introduction

Despite increasing levels of ethnic and racial diversity, racial segregation is a defining feature of American cities. According to the 2010 Census, the average urban,[1] African American household lives in a neighborhood that is only 35% white (Logan & Stults 2011).[2] Empirical evidence suggests that racial segregation affects health and education (Cutler & Glaeser 1997, Card & Rothstein 2007), labor market outcomes (Boustan & Margo 2009), and social interactions (Alesina & Ferrara 2000). Yet, while overall racial segregation across neighborhoods remains high, the racial composition of some neighborhoods changes dramatically over short periods of time.

Social interaction models explain the mechanisms of neighborhood *tipping* (Schelling 1971), whereby the entry of a small number of minority residents in a neighborhood is followed by large outflows of white households, departures that are often referred to as *white flight* (Grubb 1982, Boustan 2007, Boustan 2010). Card, Mas & Rothstein (2008) present evidence of neighborhood tipping in recent decades, where the fraction of minority residents that triggers large departures of white households ranges from 5% to 20%.[3] Saiz & Wachter (2011) show that neighborhoods that are spatially contiguous to immigrant enclaves are more likely to become relatively immigrant-dense themselves.

Historical and sociological evidence (Clark 1965, Helper 1969, Hirsch 1983, Orser 1994, Gotham 2002, Seligman 2005) as well as articles in law journals (Glassberg 1972, Moskowitz

[1]We consider a household to be *urban* when it resides in a metropolitan statistical area (MSA).

[2]According to results reported in Bayer, McMillan & Rueben (2004), segregation by socioeconomic characteristics does not fully explain racial segregation.

[3]This evidence is disputed by Easterly (2009).

1977, Mehlhorn 1998) suggest that brokers play a decisive role in neighborhood tipping. The National Association of Real Estate Boards found the issue sufficiently concerning that, until 1950, Article 34 of Part III of its Code of Ethics specified that "a realtor should never be instrumental in introducing into a neighborhood a character of property or occupancy, members of any race or nationality, or any individuals whose presence would clearly be detrimental to property values in that neighborhood."[4] Congress was also concerned about the issue, and a series of congressional hearings during development of the 1968 Civil Rights Act considered the role of real estate brokers in neighborhood change. These hearings led to section 804[e] of Title VIII of that legislation,[5] which prohibits *blockbusting*, defined as follows:[6]

> [e] For profit, to induce or attempt to induce any person to sell or rent any dwelling by representations regarding the entry or prospective entry into the neighborhood of a person or persons of a particular race, color . . . .

However, little is known about the role of real estate brokers in the process of neighborhood tipping. There is substantial evidence that brokers steer minority and white buyers and sellers to particular houses, particular households, and/or particular neighborhoods (Turner & Mikelsons 1992, Ondrich, Ross & Yinger 2003). However, the dynamic models of urban segregation (Schelling 1969, Schelling 1971, Benabou 1993, Benabou 1996, Becker & Murphy 2001, Frankel & Pauzner 2002) do not explain the mechanisms allowing brokers to change the equilibrium composition of a neighborhood.

This paper focuses on the ability and the incentives of real estate brokers to engage in blockbusting. It presents a continuous-time dynamic model of a neighborhood in racial transition as triggered by fee-motivated real estate brokers.[7] We show that racial transition

[4]The reference to "race or nationality" was removed in 1956, but the rest of the article remains in the Code of Ethics.

[5]Title VIII of the 1968 Civil Rights Act is also called the 1968 Fair Housing Act.

[6]Newspapers also reported a large number of alleged cases of blockbusting: "Real estate broker's license revoked in blockbusting case," *Chicago Tribune,* May 9, 1978; "Blockbusting laid to realtor," *New York Times,* May 19, 1973; "Scare practices in realty studied," *New York Times,* July 18, 1960.

[7]The real estate broker is an intermediary who earns commission fees. In the words of *Blockbusting* (1970):

will occur only when (i) white households have moderate racial preferences, (ii) the value of housing outside the neighborhood for minority households is low, (iii) the broker is moderately patient, (iv) the arrival rate of offers is moderate, and (v) the number of brokers is limited. In particular, the model suggests that steering white households toward black sellers does not generate more revenue if any of racial preferences, offer arrival rates, or broker impatience are extreme.

The broker decides whether to steer white sellers to minority or to white buyers.[8] This choice is based on comparing his fee revenue in an all-white neighborhood with that results if the same neighborhood experiences racial transition.[9]

We assume that white households prefer white neighbors over minority neighbors.[10] The neighborhood has a natural turnover rate that generates brokerage fees even when its racial composition does not change. When the broker matches white sellers to minority buyers, there is an increase in the fraction of minority households in the neighborhood. At first, only whites with a low valuation of the neighborhood sell to minority buyers. We show that there is a unique tipping point: a fraction of minorities in the neighborhood beyond which all white households—regardless of their valuation of the neighborhood—are ready to sell. Then turnover is high, although prices (and thus brokerage fees per transaction) are lower than in the all-white neighborhood.

The broker thus faces a trade-off. On the one hand, maintaining an all-white neighborhood ensures high prices and high brokerage fees but results in low turnover. On the

"The modern blockbuster ... serves as an agent in panic sales from white owners to black buyers and profits from the numerous commissions attendant to transforming a neighborhood."

[8] In law, Bisceglia (1973) defines blockbusting as "technically ... the first sale in a previously all white area to a black—so that the racial homogeneity of the block is 'busted'."

[9] Thus the model endogenizes the steering behavior of real estate brokers. This is in line with Aleinikoff (1976), which claims: "The quickest and surest sales can be made by satisfying buyer preferences, which brokers assume to be for neighborhoods inhabited by members of the buyer's own race." In this paper we give the conditions under which steering to a buyer of the seller's own race maximizes the broker's revenue.

[10] In contrast with Becker & Murphy (2001), we also assume that households' expectations are forward looking and not myopic. This paper is hardly the first to feature forward-looking expectations of neighborhood change. For instance, Frankel & Pauzner (2002) show that, when white households expect neighborhood change, tipping occurs earlier than in a model with myopic expectations. That paper also shows that modeling forward-looking expectations reduces the number of equilibria in specific cases, but it does not address steering by brokers.

other hand, triggering a racial transition generates sequential patterns of transactions and fees. Prices and turnover are initially low: high-valuation white households do not sell (low turnover) and white households expect a neighborhood change (low prices). But when the neighborhood reaches its tipping point, turnover increases and the broker realizes high revenues. After the transition, however, turnover and prices decline to levels below those for an all-white neighborhood. Given these dynamics, the broker's incentive to trigger "white flight" depends on whether or not the medium-term benefits of a large number of transactions offset the lower transaction fees throughout the transition and in the long run.

We analyze how this trade-off depends on various parameters of the model, beginning with white households' racial preferences. In the case of strong racial aversion, the tipping point would be reached rapidly but transaction prices would fall rapidly as well. When racial preferences are weak, the tipping point is not reached until after a long period of time characterized by low turnover and low transaction fees. It thus turns out that blockbusting generates more revenue only for a bounded range of racial preferences.

Next we consider the effect of the broker's impatience. Besides the usual time-discounting, the broker's impatience captures implicitly the likelihood that the broker no longer extracts rents from a particular neighborhood in the future (though broker entry and exit are not explicitly in the model.) Impatient brokers may be less inclined to practice blockbusting because of the lower short-term brokerage fees (before the tipping point is reached). Moderately patient brokers are willing to earn lower revenues at first as long as higher revenues can be reasonably anticipated in the medium term (i.e., for some time after the tipping point). Finally, brokers who are extremely patient do not benefit from blockbusting owing to the lower long-term brokerage fees. Hence, there is a bounded range of broker discount rate for which blockbusting generates more revenue.

We also take up the effect of minorities' value of housing outside the neighborhood. When this outside option is lower, blockbusting incentives are higher. When the options of minority households improve, transaction prices (and brokerage fees) decrease. At the same time, the tipping point occurs later because minority households have a lower relative valuation of the target neighborhood. Both effects reduce the broker's revenue. These results shed light on historical accounts that brokers' blockbusting activity generated substantial brokerage revenue in metropolitan areas where the quality of housing supplied to minority households was low.[11]

Finally, we study the comparative statics with respect to the rate of arrival of offers. A higher rate has an ambiguous effect on blockbusting revenues: although it hastens the tipping point, it also implies a more rapid entry of minorities; that, in turn, reduces prices because white households' valuation of the neighborhood incorporates forward-looking expectations of neighborhood change. For a bounded range of offer arrival rates, the broker's blockbusting revenue is higher than his revenue from a steady-state white neighborhood. For high rates of offers, blockbusting revenue is actually lower than revenue generated from a steady-state white neighborhood.

We extend the model to two brokers in order to explicitly examine the free-riding among brokers that can discourage a broker from engaging in blockbusting. We discover that if one broker matches white sellers to minorities then the other broker does, too, but only with some delay. Indeed, blockbusting initially involves lower commission fees. In equilibrium, one broker matches sellers to minority buyers all along while the other broker free-rides on the first broker's groundbreaking efforts to change the neighborhood's racial composition. In particular, the second broker initially matches sellers to white buyers and only later—when the commissions from matching to minority buyers exceed those from matching to white buyers—sells also to minorities. These results—when combined with those on the arrival rate of offers—imply that, with a large number of brokers, blockbusting revenue is less than the revenue from a steady-state white neighborhood. That is, competition discourages blockbusting.

[11] Mehlhorn (1998) cites the City Commission on Human Rights of New York: "blockbusting investigations revealed sordid patterns of racial and economic exploitation of New York City families desperately seeking decent places in which to live." In addition, Orser (1994) and Seligman (2005) describe successful blockbusting in white neighborhoods that were close to black neighborhoods with slum or low-quality housing.

The paper proceeds as follows. Section 2 presents the model. Section 3 solves the model for the cases of no blockbusting (Section 3.1) and blockbusting (Section 3.2). Section 3.3 compares the revenues derived from racially transitioning and steady-state all-white neighborhoods. Section 4 analyzes the effects of key parameters on the broker's incentives to blockbust: the effects of racial preferences, the broker's discount rate, the minorities' outside option, and the arrival rate of offers. Section 5 presents the case of two brokers: one broker free-rides on the other broker's effort to introduce minority households in the neighborhood, so a broker has less incentive to blockbust when there is another broker in the neighborhood. Section 6 concludes. Proofs for all results are given in the Appendix.

2 Model

2.1 Neighborhood and Households

The neighborhood has a continuum of houses indexed by $i \in [0, 1]$. The residents of the neighborhood are homeowning households; each house is owned by one household and each household owns one house. For each house i, we let i also designate the owner, though the identity may change over time. Homeowners are divided into two races, white ($r = w$) and black ($r = b$).[12] Whites can be either "relaxed" or "distressed", as described in Section 3.2.

Time $t \in [0, \infty)$ is continuous. The household's discount factor is β. The *flow value* for whites living in the neighborhood is $v_w - e - \rho b(t)$; here v_w is the flow value of local amenities and e is the household's type, relaxed ($e = 0$) or distressed ($e = \varepsilon$).[13] We use $b(t)$ to denote the fraction of black households that reduces the neighborhood's flow value of utility by $\rho b(t)$, where ρ is the strength of racial aversion.[14] The flow value for blacks living in the

[12] We model two racial groups, which might also be Hispanics and African Americans. The important assumption is that the incumbent race prefers neighbors of its own race.

[13] The shock be affects the utility of living in this particular neighborhood and may be due to job loss or family events. This feature is inspired by Albrecht, Anderson, Smith & Vroman (2007), who models—in a context different from racial segregation—households transitioning from a relaxed to a desperate state according to a Poisson process.

[14] Bayer, Ferreira & McMillan (2007) provide estimates of the distribution of white households' racial

neighborhood is v_b, which is independent of the fraction of blacks in the neighborhood.[15]

At $t = 0$, all houses $i \in [0, 1]$ of the neighborhood are owned by white households. At time t, the density of relaxed white households is denoted $R(t)$ while that of distressed white households is denoted $D(t)$. By construction, $R(t) + D(t) + b(t) = 1$. In a time interval $[t, t + dt)$, a relaxed (resp., distressed) white household becomes distressed (resp., relaxed) with probability $\lambda\, dt$ according to a Poisson process.

The discounted utility of living in the neighborhood for relaxed (resp., distressed) households is denoted V_{sw}^R (resp., V_{sw}^D).

Households enjoy a fixed[16] flow value v_{lr} when living outside the neighborhood, a value that depends on race $r \in \{w, b\}$. The present discounted value when living outside the neighborhood is $V_{lr} = v_{lr}/\beta$.

2.2 Matching

The housing market has several frictions. First, there is a single broker through whom all transactions must take place. Second, there are also frictions in matching the households to the broker; this is realistic but we capture these frictions in a simple way: the broker approaches anonymous homeowners in the hope of generating a sale but the households cannot approach the broker. Though the broker can observe a buyer's race, he cannot observe whether a white buyer is relaxed or distressed.

A household living in the neighborhood is matched to a broker with probability $\delta\, dt$ in a time interval $[t, t + dt)$.[17] The broker matches the household to a potential buyer. There

preferences using detailed transaction-level data in the San Francisco Bay area and assuming heterogeneity in households' preferences.

[15]The model could be generalized so that blacks prefer black neighbors (or less distaste for black than for white neighbors); strongly similar implications would follow. If the preference of black households for white neighbors is greater than that of white households for white neighbors, then the tipping point is achieved at $t = 0$ for $\hat{b} = 0$.

[16]The value of housing outside the neighborhood will be fixed if two conditions obtain: (i) there is an infinitely elastic supply of housing outside the neighborhood, so that the cost of housing is determined by the marginal cost of construction and land; and (ii) households are racially segregated outside the neighborhood.

[17]As Hsieh & Moretti (2003) point out, the vast majority of real estate transactions occur through a real estate broker. Yavaş (1992) presents a model where the real estate broker is an *additional* way to search for

exists an infinite measure of potential buyers—living outside the neighborhood—from which the broker can choose. The broker observes the race of the seller but not her state (relaxed or distressed). White households that move into the neighborhood are initially relaxed.

When a seller is matched to a potential buyer, the two households perform the transaction only if doing so is mutually beneficial. The dummy variables $\tau_{Db}(t)$, $\tau_{Dw}(t)$, $\tau_{Rb}(t)$, and $\tau_{Rw}(t)$ are set equal to 1 or 0 to indicate whether or not, respectively, distressed whites (D) sell to blacks (b), distressed whites (D) sell to whites (w), relaxed whites (R) sell to blacks (b), and relaxed whites (R) sell to whites (w) at each time $t \in [0, \infty)$.

When a transaction occurs, buyer and seller split the surplus equally according to a symmetric Nash bargaining mechanism. The respective transaction prices are denoted $p_{Db}(t)$, $p_{Dw}(t)$, $p_{Rb}(t)$, and $p_{Rw}(t)$, where $\{D, R\}$ is the white seller's state and $\{b, w\}$ is the buyer's race.

The seller and the buyer each pay a brokerage fee of $\kappa(p)$ that is linear in the transaction price: $\kappa(p) = \alpha p + \alpha_0$.[18] The broker earns revenue $\pi(t)\,dt$ per unit of time $[t, t + dt)$. His time discount factor is r, so the net present value of future revenues at time t is $\Pi(t) = \int_t^{+\infty} \pi(s)e^{-rs}\,ds$.

A key assumption is that the broker is able to choose the race of the potential buyer. In other words, the broker chooses whether to match white sellers to black or to white buyers.

The broker's strategy is assumed to take the following form. He starts by matching sellers solely to white buyers from $t = 0$ to $t = \tilde{t}$, and he begins matching sellers solely to black buyers from $t = \tilde{t}$ to $t = \infty$. If the broker never matches a white seller to a black buyer, then $\tilde{t} = \infty$.

Definition 1. An equilibrium is characterized by several factors: the fraction of black households $b(t)$ in the neighborhood at time $t \in [0, \infty)$; the fraction of distressed $D(t)$ and relaxed

housing and where the household's and broker's search efforts increase the probability δ of being matched to a buyer. In our model, the household's search effort is zero and the broker's search effort is fixed. However, this model could easily be extended to nonzero household search effort and endogenous broker search effort.

[18] We model the brokerage fee as a fixed percentage of the transaction price. This accords with Hsieh & Moretti (2003), who use the Consumer Expenditure Survey to show that the median commission fee paid was 6.1% of the transaction price and exhibited little variance.

$R(t)$ white households in the neighborhood at time t; indicator variables $\tau_{Db}(t)$, $\tau_{Dw}(t)$, $\tau_{Rb}(t)$, and $\tau_{Rw}(t)$, each set equal to 0 or 1; transaction prices $p_{Db}(t)$, $p_{Dw}(t)$, $p_{Rb}(t)$, and $p_{Rw}(t)$; and the broker's strategy $\tilde{t}$. At equilibrium, the following statements hold.

- The broker's strategy $\tilde{t}$ is the earliest moment in time at which his present discounted value of revenues when matching sellers to black buyers is higher than the present discounted value of revenues when matching sellers to white buyers.

- Sellers and matched buyers accept mutually beneficial trades, and the two parties split the transaction surplus equally.

Finally, we make one assumption regarding the initial fraction $D(t = 0)$ of distressed white households in the neighborhood.

Assumption 1. $D(0) = \lambda/(\delta + 2\lambda)$.

In the next section we show that this assumption guarantees a constant stock of distressed white households in the all-white neighborhood.

3 Steady-State All-White Neighborhood versus Blockbusting

3.1 Steady-State All-White Neighborhood

This section derives households' optimal behavior when the broker matches households with white buyers only from $t = 0$ onward.[19] Because the neighborhood is initially all-white, it remains so—that is, $b(t) = 0$ for $t \geq 0$. We shall derive the equilibrium turnover of the neighborhood and the broker's revenue.

[19]We show that Assumption 1 implies that the broker either (i) always matches sellers to white buyers (this section) or (ii) always matches sellers to black buyers (next section).

We demonstrate that the equilibrium of neighborhood depends on the size of the shock ε to the flow value v_w. Either (i) no trade occurs if ε is small or (ii) distressed households sell to whites from outside the neighborhood if ε is above a threshold noted $\tilde{\varepsilon}$.

Case #1: No Transaction

Assume first that the shock ε is insufficient to make a distressed household willing to sell. In this case, no trade occurs. Hence the Bellman equation for distressed whites' value of the neighborhood is $\beta V_{sw}^D = v_w - \varepsilon + \lambda(V_{sw}^R - V_{sw}^D)$ and for relaxed whites is $\beta V_{sw}^R = v_w + \lambda(V_{sw}^D - V_{sw}^R)$. The respective value functions are $V_{sw}^R = [v_w - \varepsilon\lambda/(2\lambda + \beta)]/\beta$ and $V_{sw}^D = [v_w - \varepsilon - \varepsilon\lambda/(2\lambda + \beta)]/\beta$, and the broker realizes no revenue.

Case #2: Relaxed Whites Sell

We assume therefore that ε is large enough to make transactions between distressed sellers and white buyers mutually beneficial. We shall derive a formula for this minimum value of ε after solving the model.

A transaction price p_{Dw} is acceptable to both seller and buyer if

$$V_{lw} + p_{Dw} - \kappa(p_{Dw}) \geq V_{sw}^D \quad \text{and} \quad V_{sw}^R - p_{Dw} - \kappa(p_{Dw}) \geq V_{lw}. \tag{1}$$

The seller and the buyer each pay the broker a commission fee $\kappa(p_{Dw}) = \alpha p_{Dw} + \alpha_0$. Hence the broker receives $2\kappa(p_{Dw})$ for each transaction. The transaction price p_{Dw} is such that the transaction generates equal gains from trade for the seller and the buyer:

$$p_{Dw} = \frac{1}{2}\left[V_{sw}^D - V_{lw} + V_{sw}^R - V_{lw}\right] \tag{2}$$

The smallest value of ε that makes trade mutually beneficial is denoted $\tilde{\varepsilon}$. This is the minimum shock ε that satisfies

$$V_{sw}^D - V_{lw} \leq V_{sw}^R - V_{lw} \tag{3}$$

We now come to finding the values of living in the neighborhood. White households' utilities satisfy the following equations:

$$V_{sw}^R = v_w\, dt + e^{-\beta\, dt}[(1-\lambda\, dt)V_{sw}^R + \lambda\, dt V_{sw}^D], \tag{4}$$

$$\begin{aligned} V_{sw}^D &= (v_w - \varepsilon)\, dt + e^{-\beta dt} \\ &\quad \times \left[\delta\, dt(V_{lw} + (1-\alpha)p_{Dw}) + (1-\delta\, dt)[(1-\lambda\, dt)V_{sw}^D + \lambda\, dt V_{sw}^R]\right]. \end{aligned} \tag{5}$$

Now expanding equations (4) and (5) with respect to dt and neglecting terms in dt^2 yields

$$\beta V_{sw}^R = v_w - \lambda[V_{sw}^R - V_{sw}^D], \tag{6}$$

$$\begin{aligned} \beta V_{sw}^D &= v_w - \varepsilon + \lambda[V_{sw}^R - V_{sw}^D] \\ &\quad + \delta[V_{lw} + (1-\alpha)p_{Dw} - \alpha_0 - V_{sw}^D]. \end{aligned} \tag{7}$$

Equations (2), (6), and (7) thus determine the values V_{sw}^R and V_{sw}^D as well as the price p_{Dw}. All three terms are decreasing in the shock ε. The Appendix shows that there is a minimum shock $\tilde{\varepsilon}$ such that condition (3) is satisfied. Above $\tilde{\varepsilon}$, a trade between a distressed white seller and a white buyer is mutually beneficial; below $\tilde{\varepsilon}$, no such trade is beneficial.

Hence, when the broker matches white households to white buyers from $t = 0$ to $t = \infty$, households' optimal behavior depends on ε. For $\varepsilon \leq \tilde{\varepsilon}$, no transaction takes place and so no white household sells. For $\varepsilon > \tilde{\varepsilon}$, distressed white households sell (but relaxed white households do not).

Proposition 1. *Assume the broker chooses to match white households to white buyers from $t = 0$ onward. When $\varepsilon \leq \tilde{\varepsilon}$, no transactions occur. When $\varepsilon > \tilde{\varepsilon}$, distressed white households* *sell but relaxed white households do not sell.*

Neighborhood Composition

Assumption 1 implies that the fraction of distressed white households $D(t)$ is constant as long as the broker matches white sellers to white buyers only. To see this, note that the change $D(t+dt) - D(t)$ in the number of distressed households in a time interval $[t, t+dt)$ is the sum of three terms:

$$\begin{aligned} D(t+dt) - D(t) &= -\delta D(t)\, dt - \lambda D(t)\, dt + \lambda R(t)\, dt \\ &= -\delta D(t)\, dt - \lambda D(t)\, dt + \lambda(1 - D(t))\, dt. \end{aligned}$$

The number of households moving into the neighborhood (who are initially relaxed) is $-\delta D(t)\, dt$; the number of distressed households staying in the neighborhood and transitioning from the distressed to the relaxed state (see Section 3.2) is $-\lambda D(t)\, dt$; and the number of relaxed households staying in the neighborhood and becoming distressed is $\lambda R(t)\, dt$.

Since $R(0) = 1 - D(0)$, the assumption $D(0) = \lambda/(\delta + 2\lambda)$ implies that the number of distressed households is constant: $D(t + dt) - D(t) = 0$. The number of transactions per unit of time is then $\delta\, dt \cdot D = \delta\lambda/(\delta + 2\lambda) \cdot dt$.

The Broker's Revenue

The broker's revenue in a period of time $[t, t + dt)$ is the sum of the commission fees (for buyer and seller) for each transaction $\delta D(t)\, dt$. At any equilibrium with transactions, only distressed households sell. Let $\pi_{\text{ss}}\, dt$ be the revenue made by the broker during each period of time $[t, t+dt)$; hence revenue π_{ss} is the number of transactions per unit of time multiplied by the average total brokerage fee per transaction: $\pi_{\text{ss}} = 2\delta D(\alpha p_{Dw} + \alpha_0)$. Therefore, we may write the present discounted value of revenues of the steady-state white neighborhood

Π_{ss} as

$$\Pi_{ss} = \frac{\delta}{r} D[2\kappa(p_{Dw})] = \frac{2\delta}{r} D[\alpha p_{Dw} + \alpha_0].$$

The transaction price p_{Dw} is given by equation (2).

The Broker's Optimal Strategy

In this section we solved the model when the broker matches sellers to white buyers only for the period from $t = 0$ to $t = \infty$. The broker's optimal strategy is either: (i) $\tilde{t} = \infty$, matching white sellers to white buyers from $t = 0$ to $t = \infty$; or (ii) $\tilde{t} = 0$, matching whites to blacks from $t = 0$ onward.

To see this point, observe that the present discounted revenue $\Pi(t)$ depends on the fraction $D(t)$ of distressed households at time t and also on the fraction $b(t)$ of black households in the neighborhood at time t. The fractions of blacks, distressed, and relaxed households are determined by the broker's strategy and by the flows to and from the distressed state. When the broker matches white sellers to white buyers, the fraction of blacks remains at $b(t) = 0$ and the fraction of distressed households is also constant, $D(t) = D(0)$. Therefore, at any time $t \leq \tilde{t}$, the broker faces the same incentives at $t > 0$ as he does at $t = 0$. From this follows our proposition that the broker, from $t = 0$ onward, will either match sellers to white buyers or match sellers to black buyers. In the next section we study the latter strategy.

3.2 Blockbusting

This section shows that, if the broker matches white sellers to black buyers from $t = 0$ onward, then either white households do not sell or households' behavior follows a tipping point equilibrium. In such an equilibrium, *distressed* whites sell from $t = 0$ onward but *relaxed* whites do not sell unless the fraction of blacks reaches the unique tipping point $\hat{b}$.

A white household meets a buyer with probability $\delta\, dt$ in the time interval $[t, t + dt)$. If the household and the buyer agree to the sale, the result is a present discounted utility of $V_{lw} + p_{Db}(t) - \kappa(p_{Db}(t))$. If seller and buyer do not agree to a sale, then the white household receives a flow value $(v_w - e - \rho b(t))\, dt$ in $[t, t+dt)$ and transitions from being relaxed (resp., distressed) to distressed (resp., relaxed) with probability $\lambda\, dt$.

When matched to a buyer, the distressed household's value of living in the neighborhood is[20]

$$\begin{aligned} V_{sw}^{D}(t) = \max\big\{ & V_{lw} + p_{Db}(t) - \kappa(p_{Db}(t)), \\ & (v_w - \varepsilon - \rho b(t))\, dt \\ & \quad + e^{-\beta\, dt}[(1 - \lambda\, dt) \cdot V_{sw}^{D}(t + dt) + \lambda\, dt \cdot V_{sw}^{R}(t + dt)]\big\} \end{aligned} \tag{8}$$

with probability $\delta\, dt$; when not matched to a buyer, the distressed household's value of living in the neighborhood is

$$\begin{aligned} V_{sw}^{D}(t) = & (v_w - \varepsilon - \rho b(t))\, dt \\ & + e^{-\beta\, dt}[(1 - \lambda\, dt) \cdot V_{sw}^{D}(t + dt) + \lambda\, dt \cdot V_{sw}^{R}(t + dt)] \end{aligned} \tag{9}$$

with probability $1 - \delta\, dt$. Analogous statements hold for the valuation $V_{sw}^{R}(t)$ of the neighborhood by a relaxed household (in which case the flow value is $(v_w - \rho b(t))\, dt$ instead of $(v_w - \varepsilon - \rho b(t))\, dt$).

Whites sell to buyers whenever there is a transaction price for which the trade is beneficial to both seller and buyer. A transaction price $p_{Db}(t)$ leads to a mutually beneficial exchange between a distressed white seller and a black buyer if both

$$V_{lw} + p_{Db}(t) - \kappa(p_{Db}(t)) \geq V_{sw}^{D}(t) \quad \text{and} \quad V_{sb} - p_{Db}(t) - \kappa(p_{Db}(t)) \geq V_{lb}.$$

[20]For simplicity, we abuse notation slightly and let $V_{sw}^{D}(t)$ and $V_{sw}^{R}(t)$ denote random variables in addition to the expected utility of living in the neighborhood at t.

Similar conditions obtain for relaxed white households selling to black buyers.

The seller and the buyer share equally in the transaction surplus via the following symmetric Nash bargaining mechanism:[21]

$$p_{Db}(t) = \frac{1}{2}\Big[(V^D_{sw}(t) - V_{lw}) + (V_{sb} - V_{lb})\Big]. \tag{10}$$

Hence prices are a linear increasing function of the relative valuation $V_{sw}(t)$ of the neighborhood.

A seller–buyer transaction proceeds whenever (after transactions costs) the seller's valuation of the neighborhood is no greater than the buyer's valuation:

$$V^D_{sw}(t) - V_{lw} \le V_{sb} - V_{lb}. \tag{11}$$

Given the price $p_{Db}(t)$ and the linearity of commission fees, condition (11) states that a distressed (resp., relaxed) white homeowner sells to a black buyer whenever the former's valuation of the neighborhood does not exceed a certain threshold, $V^D_{sw}(t) \le V_{sb} - V_{lb} + V_{lw}$ (resp., $V^D_{sw}(t) \le V_{sb} - V_{lb} + V_{lw}$).

Because the broker is seeking black buyers for white sellers, the fraction of blacks in the neighborhood is weakly increasing—that is, $b(t') \ge b(t)$ for any $t' > t$. Hence the value functions $V^D_{sw}(t)$ and $V^R_{sw}(t)$ are decreasing with time. Therefore, a homeowner (distressed *or* relaxed) who will sell at time t will also sell at any time $t' > t$.

Because distressed households experience a negative valuation shock $-\varepsilon\, dt \le 0$ during the time interval $[t, t+dt)$, it follows that the value $V^R_{sw}(t)$ of the neighborhood for relaxed whites is never less than the corresponding value $V^D_{sw}(t)$ for distressed whites. That is, $V^R_{sw}(t) \ge V^D_{sw}(t)$. So if distressed whites do not sell then relaxed whites do not sell, either.

Finally, note that if distressed whites do not sell at $t = 0$ then (a) the fraction $b(t)$ of

[21] The difference $p_{Dw} - p_{Db}(t) > 0$ is the difference between the transaction price when the broker keeps the neighborhood white vs when he does blockbusting. Newspapers reported that families whose neighborhood experienced blockbusting sold "at a sizable loss" (Vitchek 1962), $p_{Dw} - p_{Db}(t)$ in the model.

black households in the neighborhood remains zero and (b) distressed whites do not sell at any time $t > 0$.

We can now use these remarks to give a full characterization of household behavior at equilibrium as follows.

Proposition 2. *Assume that the broker matches white households to black buyers from* $t = 0$ *to* $t = \infty$. *Then there exist two values,* ε^* *and* ε^{**}, *such that the following conditions hold.*

- ***Case #1:*** *For* $0 < \varepsilon < \varepsilon^*$, *distressed whites do not sell.*
- ***Case #2:*** *For* $\varepsilon^* < \varepsilon < \varepsilon^{**}$: *either (i) distressed whites do not sell at any time* t, *and* $b(t) = 0$ *for* $t \in [0, \infty)$; *or (ii) distressed whites sell for* $t \in [0, \infty)$ *and relaxed whites sell for* $t \geq \hat{t}$, *where* $\hat{t} \in [0, \infty)$.
- ***Case #3:*** *For* $\varepsilon \geq \varepsilon^{**}$, *distressed whites sell for* $t \in [0, \infty)$ *and relaxed whites sell for* $t \geq \hat{t}$, *where* $\hat{t} \in [0, \infty)$.

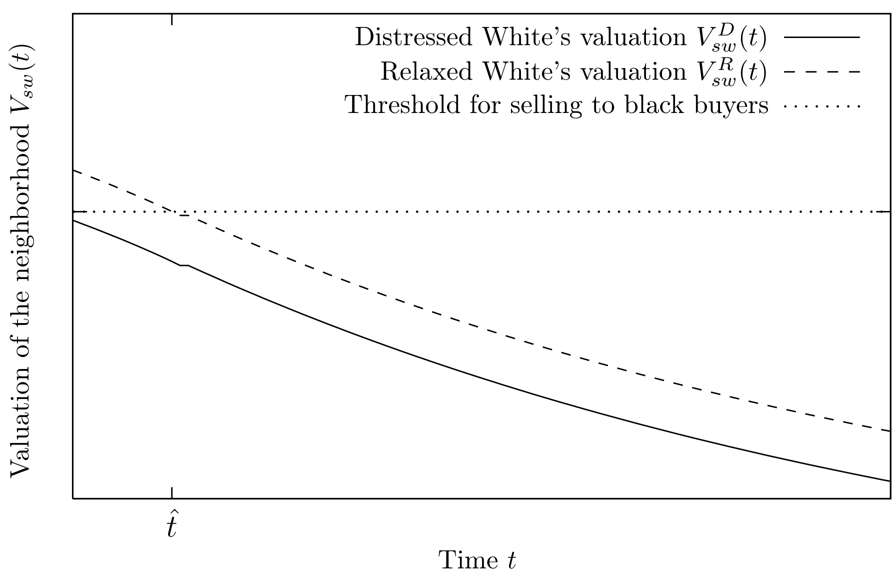
Proof. See Appendix. □

For values of $\varepsilon \in [\varepsilon^*, \varepsilon^{**}]$, distressed whites sell at $t = 0$ only if other distressed whites sell. Here ε^* and ε^{**} denote (respectively) the largest and smallest values of ε for which selling at $t = 0$ is a dominated strategy for distressed whites.

We start with the determination of ε^{**}. Selling at $t = 0$ is a dominant strategy for distressed whites if *not* selling at $t = 0$ is a dominated strategy. In turn, not selling is a dominated strategy if there is any gain from selling to a black buyer when other distressed white households do not sell:

$$\left[\frac{v_w - \varepsilon(\lambda + \beta)/(2\lambda + \beta)}{\beta}\right] \leq V_{sb} - V_{lb}. \tag{12}$$

The term $[v_w - \varepsilon(\lambda + \beta)/(2\lambda + \beta)]/\beta$ is the present value (for a distressed white household) of staying in the neighborhood when no whites sell to blacks. Hence condition (12) defines the minimum value of the shock ε^{**} as the ε that satisfies (12) with equality.



This figure presents an example of determining the time needed to reach the tipping point $\hat{t}$. Initially, at $t = 0$, distressed whites sell to blacks; that is, $V_{sw}^D(t=0) - V_l^w + \kappa(p_{Db}(t=0)) \leq V_{sb} - V_{lb} - \kappa(p_{Db}(t=0))$. At $t = 0$, relaxed whites do not sell. These households start selling as soon as they become indifferent between selling and not selling to black buyers, which determines the tipping point $\hat{t}$. In the figure, this tipping point is at the intersection of the downward-sloping dashed curve and the horizontal dotted line.
The closed-form expressions of $V_{sw}^R(t)$ and $V_{sw}^D(t)$ are given in Section 3.2. The plots are generated by a numerical simulation of the model using parameters provided in the Appendix.

Figure 1: Whites' Valuation of the Neighborhood at Time t (Broker Matches White Sellers to Black Buyers)

Next we determine ε^*, which is the smallest value of the shock for which selling is a dominated strategy when other distressed white households sell; as before, $V_{sw}^D(t)$ denotes the value of living in the neighborhood at time t given that distressed white households sell. This case prevails whenever no surplus is generated by a sale between a distressed white seller and a black buyer (conditional on other white households selling):

$$V_{sw}^D(0) \geq V_{sb} - V_{lb} + V_{lw}. \tag{13}$$

Hence, condition (13) defines the maximum value of the shock ε^* as the ε that satisfies (13) with equality.

The case $\varepsilon < \varepsilon^*$ (where no white household sells when matched to a potential buyer) is of little interest, as it yields no revenue for the broker and no change in the neighborhood's racial composition. This explains our second assumption.

Assumption 2. $\varepsilon \geq \varepsilon^*$.

Determination of the Unique Tipping Point

The second important result we show is that, in an equilibrium where white households sell to black buyers, the tipping point $\hat{b}$ is unique.

Our derivation proceeds as follows. First, at the tipping point, white sellers are indifferent between selling and not selling; thus the value of the neighborhood (relative to living outside it) should be the same for both sellers and buyers. Second, we notice that the value function is continuous at any time t—in particular, at the tipping point $\hat{t}$. We can therefore derive households' value of living in the neighborhood *at* the tipping point by using the closed-form expression for the value function at any time t *after* the tipping point.

The Bellman equation for valuing the neighborhood at time $t \geq \hat{t}$ after tipping is given by

$$
\begin{aligned}
V_{sw}^{D}(t) = \delta\, dt \cdot [V_{lw} + p_{Db}(t) - \kappa(p_{Db}(t))] \\
+ (1-\delta\, dt) \cdot \big[(v_w - \varepsilon - \rho b(t))\, dt \\
+ e^{-\beta\, dt}[(1-\lambda\, dt)\cdot V_{sw}^{D}(t+dt) + \lambda\, dt \cdot V_{sw}^{R}(t+dt)]\big]
\end{aligned} \tag{14}
$$

for distressed whites and

$$
\begin{aligned}
V_{sw}^{R}(t) = \delta\, dt \cdot [V_{lw} + p_{Rb}(t) - \kappa(p_{Rb}(t))] \\
+ (1-\delta\, dt) \cdot \big[(v_w - \rho b(t))\, dt \\
+ e^{-\beta\, dt}[(1-\lambda\, dt)\cdot V_{sw}^{R}(t+dt) + \lambda\, dt \cdot V_{sw}^{D}(t+dt)]\big]
\end{aligned} \tag{15}
$$

for relaxed whites. We show in the Appendix that $V_{sw}^{R}(t)$ and $V_{sw}^{D}(t)$ differ only by a constant. That is, $V_{sw}^{D}(t) = V_{sw}^{R}(t) - e$ for $e = \varepsilon/(\beta + 2\lambda + \frac{\delta}{2}(1+\alpha))$.

We must now find the fraction of blacks $b(t)$ at any time $t \geq \hat{t}$. The racial composition of the neighborhood is determined by the arrival rate of offers to white households and by the tipping point $\hat{b}$. When the number of blacks in the neighborhood exceeds $\hat{b}$, the change in the fraction blacks in each period $[t, t+dt)$ is $b(t+dt) - b(t) = \delta w(t)\, dt$. Since $w(t) = 1 - b(t)$, the fraction of blacks satisfies $\frac{db}{dt} = \delta(1 - b(t))$ with $b(\hat{t}) = \hat{b}$. The solution is $b(t) = 1 - (1-\hat{b})e^{-\delta(t-\hat{t})}$ for any $t \geq \hat{t}$.

The Appendix shows that, for relaxed whites at any time $t \geq \hat{t}$ (after the tipping point has been reached), the neighborhood's value is

$$
V_{sw}^{R}(t) = \mathbf{V} + e^{-\delta(t-\hat{t})}\frac{\rho(1-b(\hat{t}))}{\frac{\delta}{2}(3+\alpha)+\beta}. \tag{16}
$$

This valuation declines exponentially with time and converges to the long-run valuation of an all-black neighborhood:

$$V_{sw}^{R}(t=\infty)=\mathbf{V}=\frac{\delta\left[\frac{1-\alpha}{2}\left(V_{sb}-V_{lb}\right)-\frac{1+\alpha}{2}V_{lw}\right]+v_{w}-\rho-\lambda e}{\beta+\frac{\delta}{2}(1+\alpha)} \tag{17}$$

as $t\to\infty$ (derived in the Appendix).

Proposition 3. *Assume the broker steers white households to black buyers. In an equilibrium where distressed white households sell to black buyers, relaxed white households sell if and only if $b\geq\hat{b}$. The tipping point $\hat{b}$ is unique and is reached when relaxed white households value staying in the neighborhood as much as black households value moving into the neighborhood:*

$$\begin{aligned}
\mathbf{V}+\rho\frac{1-\hat{b}}{(3+\alpha)\frac{\delta}{2}+\beta}-V_{lw} &= V_{sb}-V_{lb}, \quad \textit{when } 1>\hat{b}>0, \\
\mathbf{V}+\rho\frac{1}{(3+\alpha)\frac{\delta}{2}+\beta}-V_{lw} &\leq V_{sb}-V_{lb}, \quad \textit{when } \hat{b}=0 \\
\mathbf{V}-V_{lw} &\geq V_{sb}-V_{lb}, \quad \textit{when } \hat{b}=1
\end{aligned} \tag{18}$$

The tipping point $\hat{b}$ is equal to 1 if relaxed white households never sell to a black buyer. That is the case when $\mathbf{V}-V_{lw}\geq V_{sb}-V_{lb}$, which could occur for one or more of the following reasons: racial preferences are not strong enough (low ρ); black buyers' relative valuation of the neighborhood is not high enough (low $V_{sb}-V_{lb}$); the prospect of becoming distressed does not have enough impact on whites' utility ($-\lambda e$); households discount the future heavily (large β); whites' valuation of the outside option is low (small V_{lw}).

The tipping point $\hat{b}$ is equal to 0 if both relaxed and distressed households sell to black buyers from $t=0$ onward. That is the case when $\mathbf{V}+\rho\big(\frac{\delta}{2}(3+\alpha)+\beta\big)^{-1}-V_{lw}\leq V_{sb}-V_{lb}$. This could happen when expectations of neighborhood change or the propsect of becoming distressed strongly influence current utility (i.e., low $\mathbf{V}$ due to a large ρ), when black buyers' relative valuation of the neighborhood ($V_{sb}-V_{lb}$) is large, or when the value of the outside option is high (large V_{lw}).

3.3 The Broker's Profit from Racial Transition

In this section we derive the revenues of a broker who steers white households in the neighborhood toward black buyers. We are ultimately interested in comparing this revenue to the revenue of the steady-state all-white neighborhood featuring the same initial conditions—that is, the same number of relaxed and distressed white households at $t = 0$.

The broker, whose time discount factor is r, collects commission fees $\pi(t)$ in each period t. If we let Π denote the net present value of future revenues at $t = 0$ and $\pi(t)\,dt$ the revenue made in a time period $[t, t + dt)$, then

$$\begin{aligned} \Pi &= \int_{t=0}^{t=\infty} \pi(t) e^{-rt}\,dt \\ &= \underbrace{\int_{t=0}^{t=\hat{t}} 2\delta D(t)\kappa(p_{Db}(t)) e^{-rt}\,dt}_{\text{Before tipping}} + \underbrace{\int_{t=\hat{t}}^{t=\infty} 2\delta[D(t)\kappa(p_{Db}(t)) + R(t)\kappa(p_{Rb}(t))] e^{-rt}\,dt}_{\text{After tipping}}. \end{aligned} \tag{19}$$

Here δ is the arrival rate of offers; $D(t)$ and $R(t)$ are (respectively) the number of distressed and relaxed households in the neighborhood at time t. As equation (19) indicates, the broker's revenue consists of two terms. The first integral term is the revenue before the tipping point has been reached, and the second is his revenue after that point has been reached.

The broker's revenue depends not only on transaction prices but also on the turnover at each moment in time. To see this, we write the turnover at time t as follows:

$$turnover(t) = \begin{cases} \delta D(t) & \text{for } t < \hat{t}, \\ \delta[D(t) + R(t)] & \text{for } t \geq \hat{t}. \end{cases}$$

Now write the average commission fees at time t as

$$commissions(t) = \begin{cases} 2\kappa(p_{Db}(t)) & \text{for } t < \hat{t}, \\ \frac{2D(t)\kappa(p_{Db}(t)) + 2R(t)\kappa(p_{Rb}(t))}{D(t)+R(t)} & \text{for } t \geq \hat{t}. \end{cases}$$

Then the present discounted value of revenues at time $t = 0$ is

$$\Pi = \int_0^\infty turnover(t) \cdot commissions(t) \cdot e^{-rt}\, dt.$$

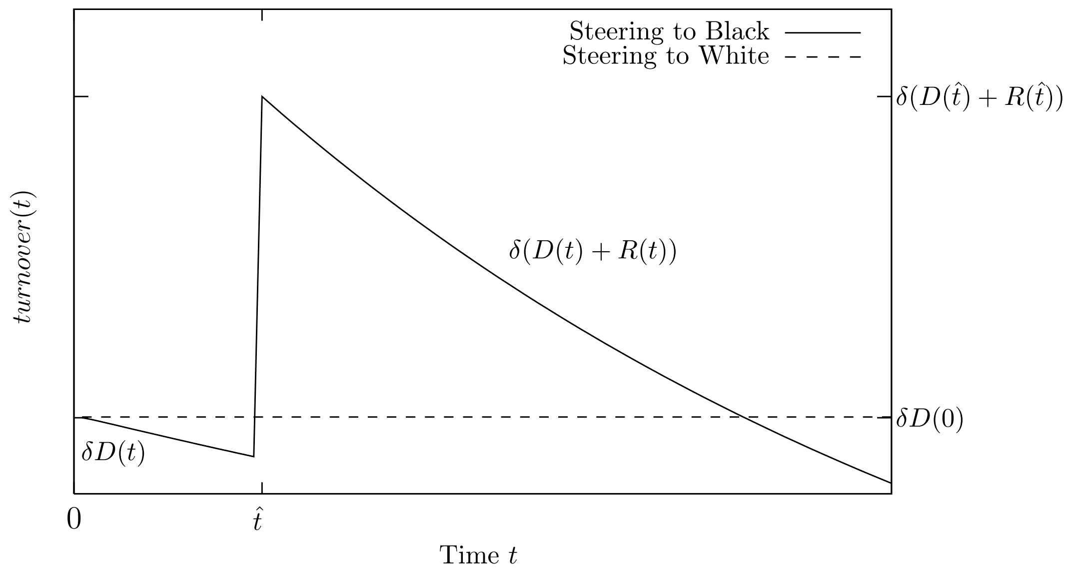
Figure 2(a) compares the turnover in racial transition and the turnover in the steady-state white neighborhood. The dynamics of the neighborhood (values of $D(t)$, $R(t)$, and $b(t)$ at each t) are derived in the Appendix. In racial transition, the turnover jumps discontinuously to $\delta\hat{w}$ at the tipping point $\hat{t}$ because, from this point onward, every matched seller agrees to the transaction. At that point, there remain $\hat{w} = 1 - \hat{b}$ whites in the neighborhood. This jump in the turnover increases the real estate broker's revenue. In contrast, the turnover is constant in the steady-state all-white neighborhood. Since the number of whites in a neighborhood converges to zero as $t \to \infty$, it follows that $D(t) \to 0$, $R(t) \to 0$, and $turnover(t) \to 0$ as $t \to \infty$.

Also, black households are not sellers. Blacks do not sell to whites because a black household has a lower valuation of the neighborhood than does any white buyer, and blacks do not sell to blacks because both parties have the same valuation.[22] Hence, as the fraction of blacks in the neighborhood approaches unity, the turnover becomes arbitrarily close to zero.

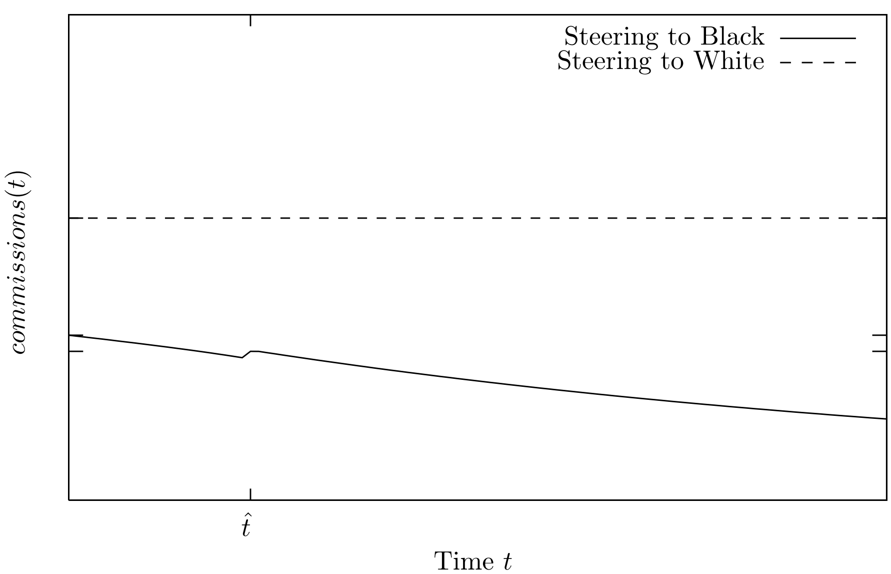
Figure 2(b) shows that average commission fees in the steady-state all-white neighborhood are higher than average commission fees in racial transition.[23] This is because transac-

[22]The reason their valuations are identical is that black households do not experience the shock ε. And since any transaction would entail commission fees, no transactions occur. Allowing for shocks to black households' valuations would be a straightforward extension of the model, whose key feature is simply that less brokerage revenue is generated in an all-black than in an all-white neighborhood.

[23]Transaction prices are given by the valuation of the neighborhood at time t. For a trade between a relaxed (resp., distressed) household and a black buyer, the transaction price depends on the valuation of the neighborhood for relaxed (resp., distressed) households. The respective closed-form values of $V^R_{sw}(t)$ and



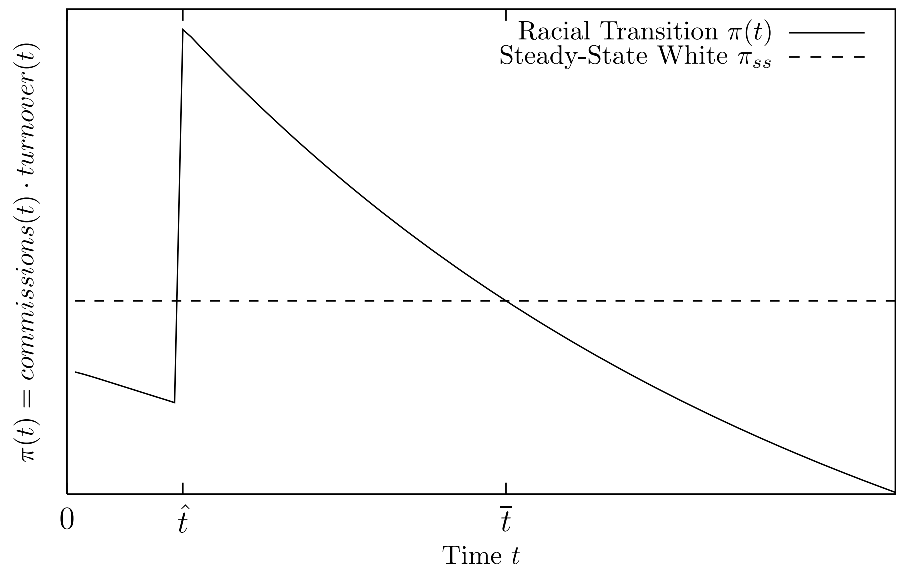
(a) Turnover at time t



(b) Turnover at time t

The plots are generated by a numerical simulation of the model using parameters provided in the Appendix.

Figure 2: Profits from Steering Sales to Black Buyers (Commission Fees and Turnover)



The plot is generated by a numerical simulation of the model using parameters provided in the Appendix.

Figure 3: Profits from Steering Sales to Black Buyers (Broker's Revenue at Time t)

tion prices in the steady-state all-white neighborhood are higher than the average transaction prices in racial transition. Once the tipping point is reached, white households expect increasing numbers of black households in the neighborhood.[24] There is a discontinuous jump in the average commission fees at the tipping point; indeed, that is when relaxed whites start accepting sales to blacks. These exchanges are transacted at higher prices than are those between black buyers and distressed white sellers.

Comparing parts (a) and (b) of Figure 2 clearly shows that the real estate broker can obtain higher revenue from a neighborhood in racial transition if the increased turnover around the tipping point compensates for the lower average commission fees. This dynamic is shown in Figure 3, which plots the instantaneous revenue $\pi(t)$ of the real estate broker at each moment in time. For the neighborhood in racial transition, the broker's revenue is lower before the tipping point. In the figure, revenue $\pi(t)$ jumps above the revenue for the

$V_{sw}^D(t)$ before and after the tipping point are given in the Appendix (see the proof of Proposition 3 and Lemma 2.

[24]Before the tipping point, the number of blacks is $b(t) = \int_0^t \delta D(s)\, ds$ (see Appendix for the closed form of this expression). After the tipping point, the number of blacks is $b(t) = 1 - (1 - \hat{b})e^{-\delta(t-\hat{t})}$.

case of the all-white neighborhood at the tipping point, and it remains above that revenue for the time period $[\hat{t}, \bar{t}]$. At $\bar{t}$, the real estate broker begins to generate less revenue in the racially transitioning neighborhood than in a steady-state all-white neighborhood.

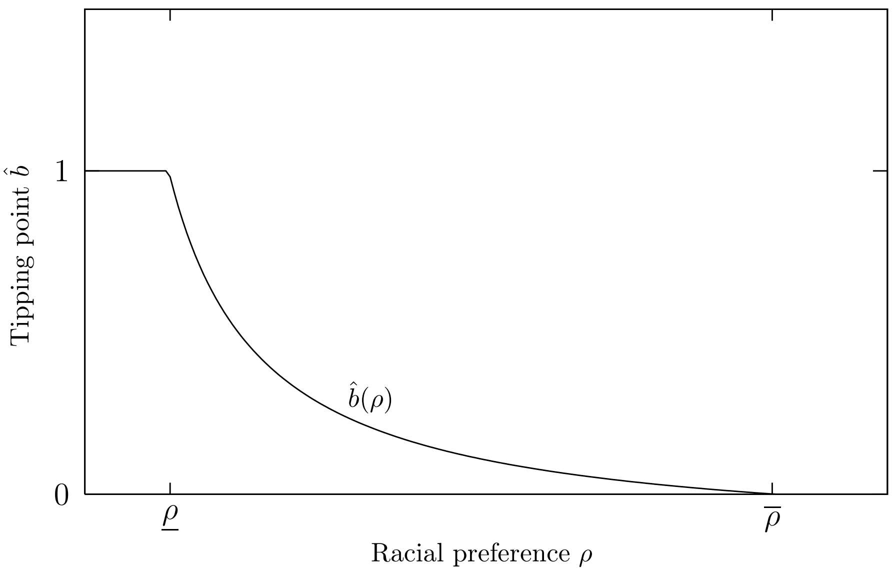
4 Implications of the Model for the Profits from Racial Transition

4.1 Racial Preferences

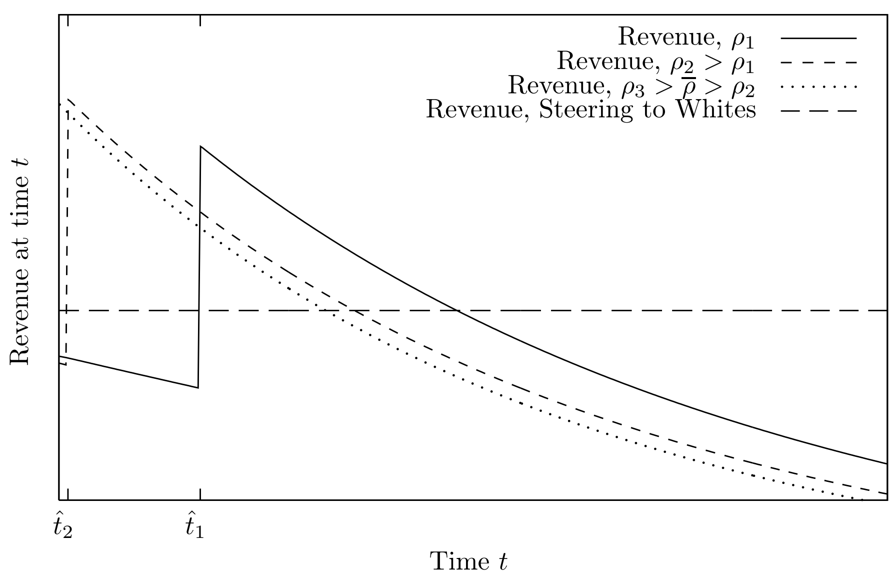
Racial preferences ρ have two effects on the broker's blockbusting revenue. First, a higher ρ (greater aversion) tends to lower the tipping point, which in turn raises broker revenue; this is the *tipping point effect.* Second, a higher ρ reduces transaction prices (and hence commission fees); this is the *price effect*, which lowers the broker's revenue.

The tipping point effect is illustrated in Figure 4(a), where the tipping point is plotted as a function of ρ. For values of ρ below $\underline{\rho}$, the tipping point is $\hat{b} = 1$ and only distressed whites sell to black households at any time t. In this case, the neighborhood's turnover and the commission fees are lower than in the steady-state all-white neighborhood; therefore, the broker's revenue is lower when steering sellers to black buyers than to white buyers. For higher values of ρ, the tipping point becomes $\hat{b} < 1$. Figure 4(b) shows a case where, at $\rho = \rho_1$, the present discounted value (p.d.v.) of broker revenue is higher in a racially transitioning neighborhood than in an all-white steady-state neighborhood (in the figure, revenue at time t is depicted by the solid line). An increase from ρ_1 to ρ_2 lowers the tipping point $\hat{b} > 0$, which increases the broker's p.d.v. of revenue from steering sellers to black buyers. However, the attendant decline in prices will decrease that p.d.v. The new curve of revenue at time t is depicted by the dashed line. The effect of ρ on total revenues is ambiguous because it depends on which of these two effects dominates.

For values of ρ above $\overline{\rho}$, the tipping point is $\hat{b} = 0$ (see Figure 4(a)). In that case, the



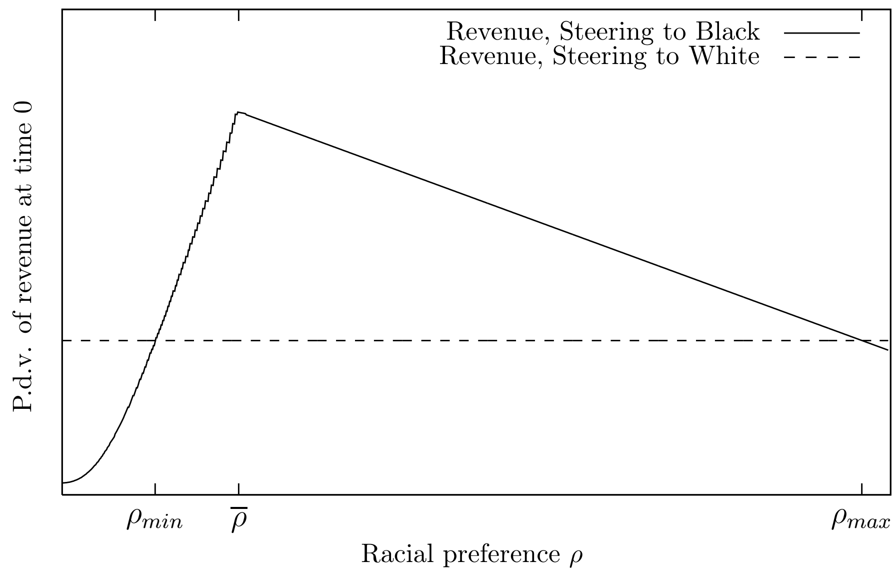
(a) The tipping point and racial preferences



(b) Revenue at time t and racial preferences

The plots are generated by a numerical simulation of the model using parameters provided in the Appendix.

Figure 4: Racial Preferences and (a) the Tipping Point, (b) Revenue at Time t



The plot is generated by a numerical simulation of the model using parameters provided in the Appendix.

Figure 5: Racial Preferences ρ and the Profits from Steering to Black Buyers

only effect is the price effect of ρ. This is illustrated in Figure 4(b) as ρ goes from ρ_2 to ρ_3. The value ρ_3 is greater than $\overline{\rho}$. At ρ_3, the only effect of an increase in ρ is a decline in average prices (and hence in average commission fees), so at this point, too, there is only a price effect. In short, the present discounted value of revenues from steering to black buyers declines with increasing ρ.

These findings are summarized in Figure 5, which plots the present discounted value of the broker's revenue at time $t = 0$ as a function of racial preferences ρ. The dashed line is the p.d.v. of the broker's revenue in the steady-state all-white neighborhood, which is independent of racial preferences. If the tipping point is $\hat{b} = 0$ ($\rho > \overline{\rho}$), then the broker's revenue is a decreasing function of racial preferences ρ. In this case, the price effect dominates. We show that revenue is a linear decreasing function of ρ for $\rho \geq \overline{\rho}$.

Proposition 4. *Profit is higher in the all-white neighborhood than in blockbusting for $\rho = 0$ and for $\rho \to \infty$. Let $\rho_{\min}$ and $\rho_{\max}$ denote, respectively, the least and most racial aversion for which broker revenue is greater under white flight than in a steady-state all-white neigh-*

borhood. Then $0 < \rho_{\min} < \rho_{\max} < \infty$. *Profit is linear and decreasing for* $\rho \geq \overline{\rho}$, *where* $\overline{\rho}$ *is the weakest level of racial preferences for which* $\hat{t} = \hat{b} = 0$.

For $\rho \leq \overline{\rho}$, the effect of an increase in ρ on broker revenue is ambiguous because the tipping point effect and the price effect are opposed: the former increases revenue whereas the latter reduces it. Numerical simulations, as graphed in Figure 5, suggest that the tipping point effect dominates the price effect for $\rho \leq \overline{\rho}$; this implies that, at time $t = 0$, the p.d.v. Π of broker revenue is an increasing function of racial preferences ρ for $\rho \leq \overline{\rho}$.

4.2 The Broker's Discount Rate

The broker's discount rate r determines the relative weight given to revenues at each stage of the neighborhood's racial transition from all white to all black. Figure 3 and Section 3.2 describe these three stages. From $t = 0$ to the tipping point $t = \hat{t}$, the broker's revenue $\pi(t)$ is *less* when steering sellers to black than to white buyers. For $t \in [\hat{t}, \bar{t}]$, the real estate broker can make *more* revenue by steering sellers to black buyers. But for $t \geq \bar{t}$, steering to blacks once again yields lower $\pi(t)$.

A high time discount rate r (i.e., an impatient broker) gives greater weight to early revenues $\pi(t)$, $t \leq \hat{t}$, than to medium-term revenues, $t \in [\hat{t}, \bar{t}]$, and long-term revenues, $t \geq \bar{t}$. Conversely, a low time discount rate r (i.e., a patient broker) gives greater weight to long-term revenues than to medium- and short-term revenues. The discount rate r thus determines a broker's incentive to blockbust, as is stated formally in the following proposition.

Proposition 5. *If the tipping point is* $\hat{b} > 0$, *then there are two values* ($\underline{r} > 0$ *and* $\overline{r} > 0$) *of the time discount factor such that, when* $r \notin [\underline{r}; \overline{r}]$, *the p.d.v. of broker revenue* $\Pi = \int_0^\infty \pi(t)e^{-rt}\,dt$ *when steering to blacks is less than the p.d.v. of revenue* π_{ss}/r *when steering to whites.*

If the tipping point is $\hat{b} = 0$, *then a broker earns more revenue from steering to black than to white buyers. That is, there exists a finite* $\overline{r}$ *such that the p.d.v. of revenues from steering* *to blacks,* $\int_0^\infty \pi(t)e^{-rt}\,dt$*, is more (resp., less) than the p.d.v. of revenues from steering to whites,* π_{ss}/r*, for* $r > \overline{r}$ *(resp.,* $r < \overline{r}$*).*

Proof. See Appendix. □

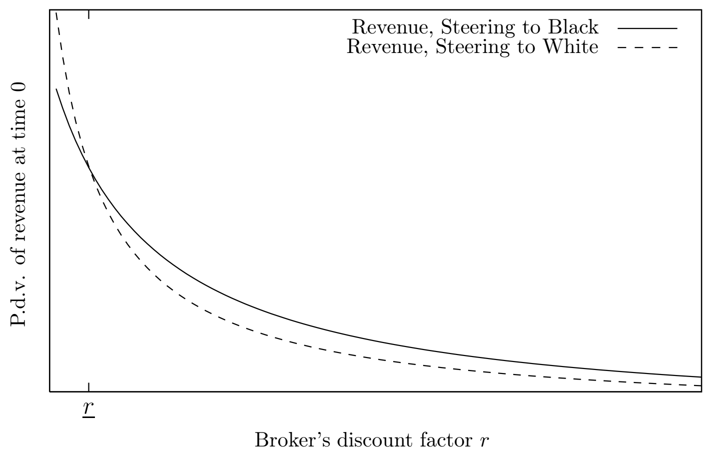
The results of this proposition are illustrated in Figure 6, where panels (a), (b), and (c) present three different scenarios. In each scenario we fix all parameters except r, which varies from zero to infinity.

In panels (b) and (c), the tipping point is nonzero. In panel (b), for values of r below $\underline{r}$ (a patient broker), the p.d.v. of revenue from steering to whites is higher than that from steering to blacks. The reason is that, for low values of r, this broker's present discounted value gives more weight to long-term revenues, which are lower if the broker steers sellers to black buyers. For $r > \overline{r}$ (an impatient broker), the revenue from steering to whites is greater than that from steering to blacks. This is because the p.d.v. of an impatient broker's revenue gives more weight to the period $t \in [0, \hat{t}]$ (i.e., before the tipping point), when turnover and prices are both low. In panel (c), the tipping point is nonzero but there is no time discount factor that leads to more revenue when steering to black buyers.

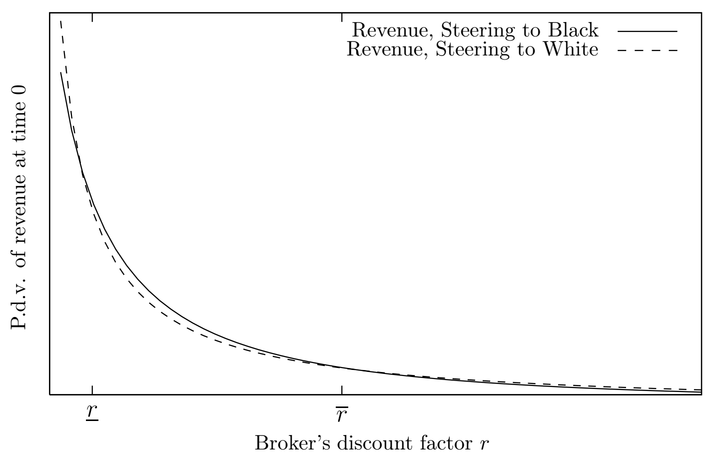
Panel (a) of Figure 6 presents a scenario where the tipping point $\hat{b} = 0$. Here the broker's revenue when steering to blacks is higher from $t = 0$ to $t = \bar{t}$. In this case, an impatient (high-r) broker always has an incentive to steer white sellers to black buyers. A patient broker ($r < \underline{r}$) values long-term revenues the most, and these are higher when the broker steers sellers to white than to black buyers. So in this case we write $0 < \underline{r} < \overline{r} = \infty$.

4.3 Black Households' Outside Option

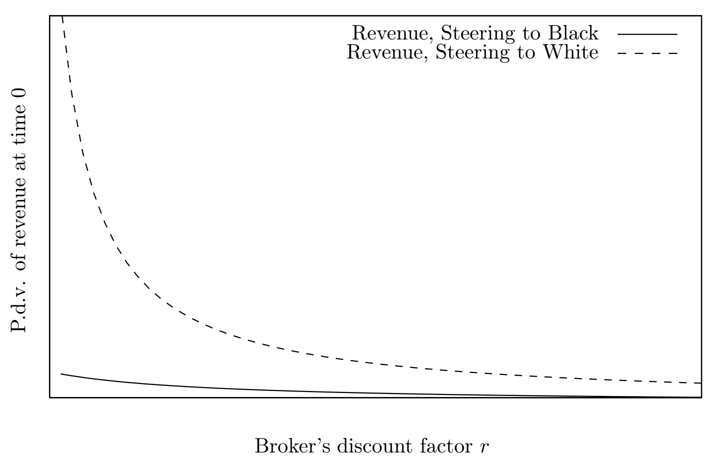
Black buyers' value of the outside option affects the broker's incentive to blockbust in a simple way. The value of living outside the neighborhood for black households is V_{lb}. When this value increases, the tipping point $\hat{b}$ and the time needed to reach that point also increase. Indeed, with higher V_{lb}, the relaxed white households' value of living in the neighborhood



(a) Case 1: Tipping point is zero, $\pi(0) > \pi_{ss}$. One intersection



(b) Case 2: Tipping point is nonzero, $\pi(0) < \pi_{ss}$. Two intersections.



(c) Case 3: Tipping point is nonzero, $\pi(t) < \pi_{ss}$. No intersection.

The plots are generated by a numerical simulation of the model using parameters provided in the Appendix.

Figure 6: Broker's Discount Factor r and the Profits from Steering to Black Buyers

needs to decline by a larger amount in order to reach the time t at which selling to blacks yields more revenue than does selling to whites. Note also that, when V_{lb} increases, transaction prices in the neighborhood decline. Hence, the p.d.v. of future revenues is decreasing in V_{lb}. Formally, we have the following result.

Proposition 6. *Transaction prices are a decreasing function of black households' outside option; that is, $dp_{Db}(t)/dV_{lb} < 0$ and $dp_{Rb}(t)/dV_{lb} < 0$. The tipping point is an increasing function of black households' outside option; that is, $d\hat{b}/dV_{sb} > 0$ and $d\hat{t}/dV_{sb} > 0$.*

The revenue from matching white sellers to black buyers is a decreasing function of blacks' valuation of housing outside the neighborhood. There is a value $\overline{V_{lb}}$ such that, for $V_{lb} < \overline{V_{lb}}$, steering white sellers to black buyers leads to more revenue than does steering them to white buyers.

Proof. See Appendix. □

4.4 Arrival Rate of Offers

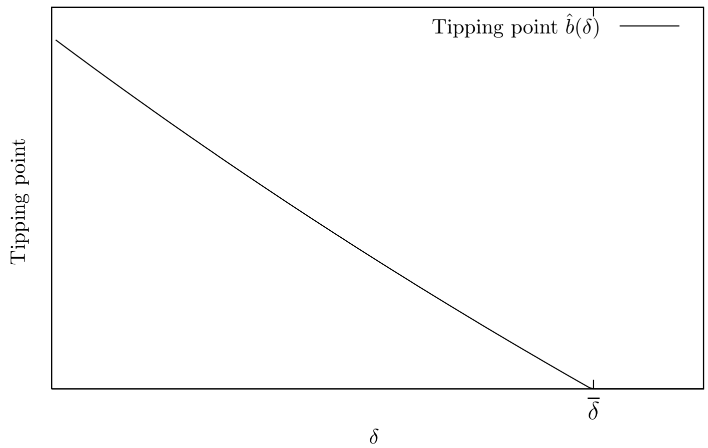
The rate δ of arrival of offers has an ambiguous effect on the broker's blockbusting revenue. In the first place, a higher arrival rate decreases the time $\hat{t}$ required to reach the tipping point, as more black households enter the neighborhood in each period $[t, t + dt)$; this increases the broker's revenue. The effect is illustrated in Figure 7(a), which shows the tipping point as a function of δ while holding all other parameters constant. We use $\bar{\delta}$ to denote the smallest value of δ for which the tipping point is zero.

Second, a higher rate of arrival of offers lowers the tipping point $\hat{b}$. Even though the higher arrival rate increases the long-term valuation $\mathbf{V} = V^R_{sw}(t = \infty)$ of living in an all-black neighborhood, households have forward-looking expectations of neighborhood change and so a higher δ has a greater (negative) impact on the current value of living in the neighborhood. Thus, the tipping point $\hat{b}$ is lower when the arrival rate of offers is higher. This effect, too, increases the broker's blockbusting revenue.

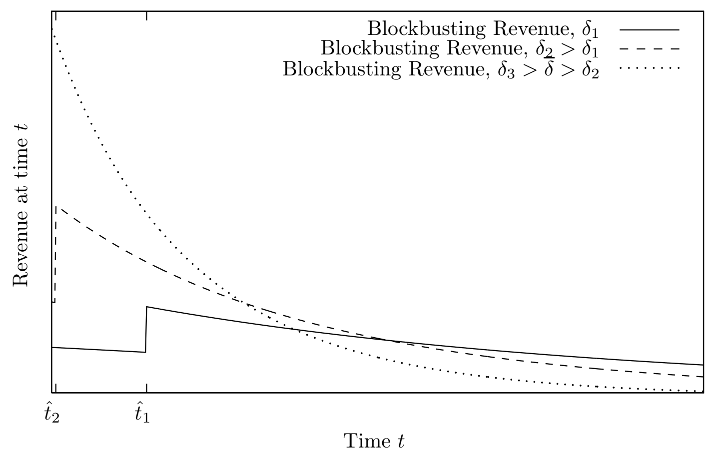
However, a third effect of higher δ is one that reduces broker revenue. Namely, households' forward-looking expectations of neighborhood change imply that the value of living in the neighborhood is lower when black households enter the neighborhood more rapidly. Hence transaction prices are lower when the arrival rate δ is higher.

These three effects are illustrated in Figure 7(b), which shows the revenue at time t for three different values $\delta_1 < \delta_2 < \delta_3$. If the offer arrival rate increases from $\delta = \delta_1$ to $\delta = \delta_2$, then the time until the tipping point is reduced from $\hat{t}_1$ to $\hat{t}_2$ and the neighborhood turnover increases. However, as $t \to \infty$, the revenue for $\delta = \delta_2$ becomes lower than the revenue for $\delta = \delta_1$ because the fraction of whites in the neighborhood is smaller in the case of a high arrival rate and so transaction prices are lower. At $\delta = \delta_3$ the tipping point is zero and so increasing δ can no longer affect the tipping point. However, increasing δ will continue to induce falling transaction prices.

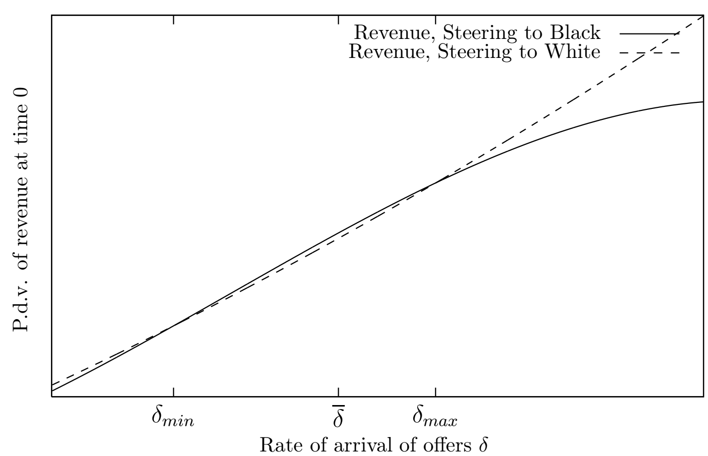
Figure 7(c) compares the present discounted value of the blockbusting revenue at $t = 0$ to the present discounted value of revenues from an all-white neighborhood. In Section 3.1 we showed that the p.d.v. of revenues in an all-white neighborhood is an increasing function of the offer arrival rate δ. For very low values of δ, the p.d.v. of the revenue from blockbusting is less than the p.d.v. of revenue from maintaining an all-white neighborhood. The reason is that the time to reach the tipping point approaches infinity as the offer arrival rate approaches zero. For very high values of δ, the p.d.v. of the blockbusting revenue is also less. This is because, as $\delta \to \infty$, white households anticipate immediate neighborhood change and so transaction prices are low. For values of $\delta \in [\delta_{\min}, \delta_{\max}]$, the revenue from blockbusting revenue is higher than from the all-white neighborhood. The difference between these two revenue levels decreases from $\bar{\delta}$ onward, the point at which δ ceases to have any effect on the tipping point.



(a) Tipping point



(b) Revenue at time t for $\delta_1 < \delta_2 < \delta_3$



(c) Revenue from blockbusting and from Steady-State All-White Neighborhood for $\delta \in [0, \infty)$

The plots are generated by a numerical simulation of the model using parameters provided in the Appendix.

Figure 7: Offer Arrival Rate δ and the Profits from Steering to Black Buyers

5 Two Brokers

We now extend the model to feature two brokers. This creates a new aspect of the model in that broker 1 and 2 interact strategically: one broker's actions have an effect on the other broker's revenues and incentives. After presenting the equilibrium, we show that the second broker free-rides on the blockbusting efforts of the first broker.

5.1 Extended Theoretical Framework

Following Section 2, we extend the model to include two brokers indexed by $j = 1, 2$. A household of the neighborhood is matched to broker $j = 1, 2$ with probability $\delta\, dt$ in period $[t, t + dt)$.

At time t, each broker decides whether to steer white sellers to black or to white buyers. We focus on brokers' monotonic strategies: for each $j = 1, 2$, we assume that broker j matches sellers to white buyers for $t \leq \tilde{t}_j$ and to black buyers for $t \geq \tilde{t}_j$. We denote by $\Pi_{2,w}(t)$ (resp., $\Pi_{2,b}(t)$) the present discounted value of broker 2's future revenues from t to ∞ when he steers sellers exclusively to white (resp., black) buyers.

Definition 2. An equilibrium with two brokers is characterized by: the fraction of black households $b(t)$ in the neighborhood at time $t \in [0, \infty)$; the fraction of distressed $D(t)$ and relaxed $R(t)$ white households in the neighborhood at time t; indicator variables $\tau_{Db}(t)$, $\tau_{Dw}(t)$, $\tau_{Rb}(t)$, and $\tau_{Rw}(t)$, each set equal to 1 (or 0) when there is (or is not) a transaction between distressed or relaxed sellers and black or white buyers, as applies; the respective transaction prices $p_{Db}(t)$, $p_{Dw}(t)$, $p_{Rb}(t)$, and $p_{Rw}(t)$; and the strategy $\tilde{t}_j$ of each broker $j = 1, 2$. At equilibrium, the following statements hold.

- The broker's strategy $\tilde{t}_j$ is the earliest time at which his p.d.v. of revenues at time $\tilde{t}_j$ when matching sellers to black buyers is greater than the p.d.v. of revenues (at that time) when matching sellers to white buyers—given the other broker's strategy $\tilde{t}_{-j}$.

- Sellers and matched buyers accept mutually beneficial trades, and the two parties split the transaction surplus equally.

5.2 Brokers' Behavior at Equilibrium

We next turn to analyzing brokers' behavior at equilibrium. Much as in the case with one broker, in any equilibrium of the model we have either that all brokers steer sellers to white buyers or that at least one of the two brokers starts steering sellers to black buyers at $t = 0$. When both brokers steer sellers to white buyers, there is no change in the number of distressed or relaxed whites (as a consequence of Assumption (1)). Hence the brokers have the same incentives to steer sellers toward white (or black) buyers at $t > 0$ as they do at $t = 0$.

What remains to be determined is the behavior of broker 2. By definition of the equilibrium, at every time t broker 2 compares the revenue from steering sellers to white buyers for *another* interval of time, $[t, t + dt)$, with the revenue from steering sellers to black buyers. The broker thus compares the following two equations:

$$\Pi_{2,w}(t) = \max\{\pi_{2,w}(t)\, dt + e^{-r\,dt}\Pi_{2,w}(t + dt), \Pi_{2,b}(t)\}, \tag{20}$$

$$\Pi_{2,b}(t) = \pi_{2,b}(t)\, dt + e^{-r\,dt}\Pi_{2,b}(t + dt). \tag{21}$$

Equation (20) gives the present discounted value of revenues from the broker steering sellers to white buyers until time t and then deciding to steer sellers to black buyers from time t onward. Equation (21) gives the p.d.v. of broker revenue when steering to blacks from t onward. Given broker 1's equilibrium behavior, we use $\pi_{2,w}(t)\, dt$ (resp., $\pi_{2,b}(t)\, dt$) to denote the fees collected by broker 2 in $[t, t+dt)$ when steering sellers to white (resp., black) buyers.

Intuitively, the point at equilibrium when broker 2 starts steering sellers to black buyers is the $\tilde{t}_2$ at which the brokerage fees $\pi_{2,w}(\tilde{t}_2)$ and $\pi_{2,b}(\tilde{t}_2)$ are equal. See Appendix for the proof, which relies on deriving closed-form solutions for broker 2's program to take the maximum of equations (20) and (21). Thus we have our final proposition.

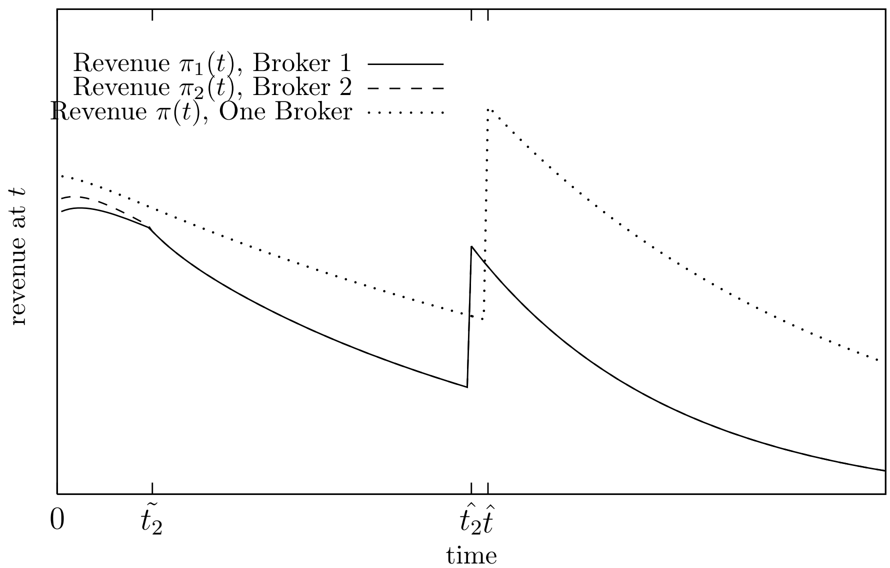
Proposition 7. *In the case of two brokers, there can be one of two equilibria as follows.*

1. *(i) Both brokers match sellers to white buyers, and neither broker matches sellers to black buyers;* $\tilde{t}_1 = \tilde{t}_2 = \infty$.
2. *(ii) One broker (say, broker 1) starts matching sellers to blacks at* $t = 0$ *(i.e.,* $\tilde{t}_1 = 0$*). The other broker (say, broker 2) matches sellers to white buyers as long as the commission fees from doing so exceed those earned from matching to black buyers; hence* $\tilde{t}_2$ *is the earliest time at which both* $\pi_{2,w}(\tilde{t}_2) = \pi_{2,b}(\tilde{t}_2)$ *and* $\pi_{2,w}(t) \geq \pi_{2,b}(t)$ *for* $t \leq \tilde{t}_2$.

5.3 Blockbusting Revenue with One and Two Brokers

With two brokers, the offer arrival rate doubles from δ to 2δ. The impact of an increase in this rate was described in Section 4.4; such an increase leads to a lower tipping point but also to lower prices/commissions. In addition to these effects, with two brokers the strategic behavior of the one who does not start blockbusting at $t = 0$ tends to lower the revenue of the one who does engage in blockbusting at $t = 0$. Indeed, broker 2 generally starts matching sellers to black buyers not at $t = 0$ but rather at $\tilde{t}_2 \geq 0$. However, if $t \geq \tilde{t}_2$ then broker 2 continues matching white sellers to black buyers.

Figure 8 shows the revenue at time t in the case of two brokers and in the case of one broker. Here the equilibrium is such that broker 1 steers sellers to black buyers from $t = 0$ onward and broker 2 does so from $t = \tilde{t}_2$ onward. The figure plots the revenue of broker 1 and broker 2 ($\pi_1(t)$ and $\pi_2(t)$, respectively) at time t; it also plots the revenue $\pi(t)$ when there is only one broker (but with the same values of the parameters).



$\tilde{t}_2$: time at which broker 2 starts matching sellers to black buyers.
$\hat{t}_2$: time to tipping point with two brokers.
$\hat{t}$: time to tipping point with one broker.
All curves were generated using the same values of the parameters.

The plots are generated by a numerical simulation of the two-broker and the one-broker model using parameters specified in the Appendix.

Figure 8: Blockbusting Revenue at Time t with One Broker and with Two Brokers

5.4 Revenue and a Large Number of Brokers

Section 4.4 showed for high offer arrival rates that, as $\delta \to \infty$, blockbusting revenue becomes less than the revenue from a steady-state all-white neighborhood.

Similarly, with a large number N of brokers, the rate of arrival of brokers is $N\delta$ and, as $N \to \infty$, broker revenue is less from racially transitioning than from the steady-state all-white neighborhoods. This dynamic is shown in Figure 7(c). Because N is large, the tipping point becomes $\hat{b} = 0$ (see Figure 7(a)). When δ increases for $\hat{b} = 0$, prices decline as the number of brokers increases and so the revenue of each individual broker declines.

6 Conclusion

This paper focuses on the ability and incentives of a real estate broker to change the equilibrium racial composition of a neighborhood. We show that the broker can turn an all-white neighborhood into an all-black neighborhood if there are enough distressed white sellers to which he can match black buyers. Once the neighborhood contains a threshold fraction of black households, all white households (whether relaxed or distressed) sell and turnover is high. The broker has a financial incentive to change a neighborhood's racial composition if doing so yields a greater present discounted value of revenues than does maintaining a stable all-white neighborhood.

The mechanisms described here suggest that we adopt a nuanced view of the broker's incentives to steer sellers to black buyers. Neighborhoods where white households strongly dislike black neighbors are, in fact, not the best candidates because prices in such neighborhoods are significantly lower when households anticipate a large increase in the black population. The broker refrains from blockbusting also in neighborhoods where white households have no particular racial preference concerning their neighbors. Rather, the broker engages in blockbusting only within a (limited) range of racial preference values (ρ) for which steering to black buyers leads to greater present discounted revenue. With respect to such racial preferences, the United States District Court for the northern district of Illinois states:

> Blockbusting and panic peddling are real estate practices in which brokers encourage owners to list their homes for sale *by exploiting fears of racial change* within their neighborhood.[25]

Fears of racial change (i.e., racial preferences) are key to the blockbusting process. However, they lead to lower valuations of the neighborhood by white households and hence to lower transaction prices and commissions. These effects may discourage blockbusting and encourage brokers to steer sellers toward white buyers.

The model also demonstrates why an impatient broker is less likely to be a blockbuster—namely, because of the initially lower fees generated thereby. Extremely high rates of arrival of offers are also unlikely to encourage blockbusting because then both short- and long-term transaction prices are lower. Extending the model to the case of two brokers indicates that the second broker free-rides on the first broker's initial blockbusting efforts. Finally, blockbusting occurs only with a limited number of brokers. When there are too many brokers, blockbusting incentives decline because then the offer arrival rate is high and so prices are low.

More generally, this paper models a broker who can choose the market equilibrium in a setting where social interactions among agents imply multiple equilibria. Hence the model should have broader applicability and relevance beyond the issue of blockbusting and white flight. Finally, because households' expectations are forward looking, the model could also be used to explain how a real estate broker can "gentrify" an all-black neighborhood by steering black households toward white buyers.

[25] Pearson v. Edgar, 965 F. Supp. 1104, 1108–09 (N.D. Ill. 1997), cited in Mehlhorn (1998).

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read:

Symbol	Meaning
α	Fraction of the price paid as commission fee by buyer and seller
δ	Rate of arrival of offers
β	Households' time discount factor
r	Broker's time discount factor
$\kappa(p)$	Brokerage fee
$b(t)$	Fraction black in the neighborhood at time t
$V_{sw}^D(t), V_{sw}^R(t)$	Present value of the neighborhood at t for (distressed, relaxed) white households
V_{sb}	Present value of the neighborhood at t for black households
$v_w + e - \rho b(t)$	Flow value of the neighborhood in $[t, t + dt)$
e	Household's type, relaxed ($e = 0$) or distressed ($e = -\varepsilon$)
ρ	Disutility of an all-black neighborhood for white households
V_{lw}	Value of living outside the neighborhood for whites
V_{lb}	Value of living outside the neighborhood for blacks
v_b	Instantaneous utility of living in the neighborhood in $[t, t + dt)$ for blacks
$\pi(t)$	Broker's profit in $[t, t + dt)$ from racially transitioning neighborhood
π_{ss}	Broker's profit from steady-state all-white neighborhood
Π	Present value of broker's revenue from racially transitioning neighborhood
Π_{ss}	Present value of broker's revenue from steady-state all-white neighborhood
$\hat{b}$	Tipping point (fraction of black households in the neighborhood
$\hat{t}$	Time to reach the tipping point
$\overline{\rho}$	Minimum racial preference for which $\hat{b} = 0$ and $\hat{t} = 0$
ε^{**}	Minimum ε for which selling is a *dominant* strategy (matched to blacks)
ε^{*}	Maximum ε for which selling is a *dominated* strategy (matched to blacks)
$\tilde{\varepsilon}$	Minimum shock for which selling is an equilibrium (matched to whites)

Table 1: Glossary of Notation for One-Broker Model

Appendix: Proofs

Steady-State All-White Neighborhood

Proposition 1. *Assume the broker chooses to match white households to white buyers from $t = 0$ onwards. For $\varepsilon \leq \tilde{\varepsilon}$, no transaction takes place. No white household sells. For $\varepsilon > \tilde{\varepsilon}$, distressed white households sell. Relaxed white households do not sell.*

Proof. Bellman equations (6) and (7) rewrite as:

$$(\beta + \lambda)V_{sw}^R - \lambda V_{sw}^D = v_w \tag{22}$$

$$-(\lambda + \frac{\delta}{2}(1-\alpha))V_{sw}^R + (\beta + \lambda + \delta + \frac{\delta(1-\alpha)}{2})V_{sw}^D = v_w - \varepsilon + \delta V_{lw} - \delta\alpha_0 \tag{23}$$

using the equilibrium transaction price $p_{Dw} = \frac{1}{2}[\frac{1}{1-\alpha}[V_{sw}^D - V_{lw} + \alpha_0] + \frac{1}{1+\alpha}[V_{sw}^R - V_{lw} - \alpha_0]]$. From equations (22) and (23) we see that V_{sw}^R and V_{sw}^D are linear functions of ε. For a change $d\varepsilon$, holding all parameters constant, the change $dV_{sw}^R/dV_{sw}^D = \frac{\lambda}{\beta+\lambda} < 1$ (from equation 22), which means that relaxed whites' valuation of the neighborhood decreases by less than distressed whites' valuation of the neighborhood for a given increase in ε. Also, we note that $V_{sw}^R \to -\infty$ when $\varepsilon \to +\infty$, $V_{sw}^R \to +\infty$ when $\varepsilon \to -\infty$, $V_{sw}^D \to -\infty$ when $\varepsilon \to +\infty$, and $V_{sw}^D \to +\infty$ when $\varepsilon \to -\infty$. Hence by the intermediate value theorem, there is $\tilde{\varepsilon} \in (-\infty, +\infty)$ such that:

$$V_{sw}^D - V_{lw} \leq V_{sw}^R - V_{lw} \qquad \text{if and only if } \varepsilon \geq \tilde{\varepsilon}$$

□

Tipping Point Equilibrium

Proposition 2. *Assume that the broker matches white households to black buyers from $t = 0$ to $t = \infty$. For $0 < \varepsilon < \varepsilon^*$, distressed whites do not sell. For $\varepsilon^* < \varepsilon < \varepsilon^{**}$, either (i) distressed* *whites do not sell at any time t, and $b(t) = 0$ for $t \in [0, \infty)$, or (ii) distressed whites sell for $t \in [0, \infty)$, and relaxed whites sell for $t \geq \hat{t}$, where $\hat{t} \in [0, \infty)$. For $\varepsilon \geq \varepsilon^{**}$, distressed whites sell for $t \in [0, \infty)$, and relaxed whites sell for $t \geq \hat{t}$, where $\hat{t} \in [0, \infty)$.*

Proof. We show here that there are only two kinds of equilibria: (i) distressed whites do not sell at any time t, and $b(t) = 0$ for $t \in [0, \infty)$, or (ii) distressed whites sell for $t \in [0, \infty)$, and relaxed whites sell for $t \geq \hat{t}$, where $\hat{t} \in [0, \infty)$. The proof relies on three properties:

- If a distressed or relaxed white sells at $t \geq 0$, he sells at any point $t' \geq t$. Hence households' strategies take the following shape $\tau_{Db}(t) = I(t \geq \hat{t}^D)$, $\tau_{Rb}(t) = I(t \geq \hat{t}^R)$ with $\hat{t}^D, \hat{t}^R \in [0, \infty) \cup \{+\infty\}$.

- If relaxed whites sell at t, then distressed whites sell at t as well. Hence, $\hat{t}^R \geq \hat{t}^D$.

- If distressed whites do not sell at $t = 0$, they do not sell at any point in time $t' > 0$. Hence, at equilibrium $\hat{t}^D = 0$ or $\hat{t}^D = +\infty$.

Hence the only two possible kinds of equilibria. We note $\hat{t} \equiv \hat{t}^R$. The uniqueness of $\hat{t}$ is proven in the next proposition.

Formulas for ε^* and ε^{**} are given in the main body of the text. $\square$

Proposition 3. *Assume $\varepsilon > \varepsilon^*$, then relaxed white households sell whenever $b \geq \hat{b}$. The tipping point $\hat{b}$ is unique and reached when the relative value of staying in the neighborhood for relaxed households is equal to the relative value of moving into the neighborhood for blacks:*

$$
\begin{aligned}
\mathbf{V} + \rho\frac{1-\hat{b}}{3\frac{\delta}{2}+\beta} - V_{lw} &= V_{sb} - V_{lb}, \quad \textit{if } 1 > \hat{b} > 0, \\
\mathbf{V} + \rho\frac{1}{3\frac{\delta}{2}+\beta} - V_{lw} &\leq V_{sb} - V_{lb}, \quad \textit{if } \hat{b} = 0 \\
\mathbf{V} - V_{lw} &\geq V_{sb} - V_{lb}, \quad \textit{if } \hat{b} = 1
\end{aligned}
$$

where $\mathbf{V}$ was defined earlier.

Proof. When the tipping point has been reached, both relaxed and distressed whites sell. The fraction of black neighbors increases such that $\frac{db}{dt} = \delta(1 - b(t))$.

The Bellman equations for the value functions are:

$$\begin{aligned} V_{sw}^R(t) &= (v_w - \rho b(t))dt \\ &+e^{-\beta dt}[\delta dt\,[V_{lw} + (1-\alpha)p_{Db}(t) - \alpha_0] \\ &+(1-\delta dt)[(1-\lambda dt)\cdot V_{sw}^R(t+dt) + \lambda dt \cdot V_{sw}^D(t+dt)]] \end{aligned} \tag{24}$$

$$\begin{aligned} V_{sw}^D(t) &= (v_w - \varepsilon - \rho b(t))dt \\ &+e^{-\beta dt}[\delta dt\,[V_{lw} + (1-\alpha)p_{Db}(t) - \alpha_0] \\ &+(1-\delta dt)[(1-\lambda dt)\cdot V_{sw}^D(t+dt) + \lambda dt \cdot V_{sw}^R(t+dt)]] \end{aligned} \tag{25}$$

Buyer and seller split the surplus of the trade equally. Hence the price at time t:

$$p_{Db}(t) = \frac{1}{2}\left[\left(V_{sw}^D(t) - V_{lw}\right) + (V_{sb} - V_{lb})\right]$$

for distressed white households, and similarly for relaxed white households. The end of the proof requires the following lemma.

Lemma 1. The difference $\Delta V_{sw}(t) = V_{sw}^R(t) - V_{sw}^D(t)$ is constant and $\Delta V_{sw}(t) = \frac{\varepsilon}{\beta + 2\lambda + \frac{\delta}{2}(1+\alpha)}$.

Proof. Define $\Delta V_{sw}(t) = V_{sw}^R(t) - V_{sw}^D(t)$, as well as $\Delta p(t) = p_{Rb}(t) - p_{Db}(t)$. Then, take the difference of equations 24 and 25. The difference satisfies:

$$\begin{aligned} \Delta V_{sw}(t) &= \varepsilon dt \\ &+e^{-\beta dt}[\delta dt(1-\alpha)\Delta p(t) \\ &+(1-\delta dt)[(1-\lambda dt)\cdot \Delta V_{sw}(t+dt) - \lambda dt \cdot \Delta V_{sw}(t+dt)]] \end{aligned}$$

Noticing that $\Delta p(t) = \frac{1}{2}\Delta V_{sw}(t)$, then

$$\begin{aligned}\Delta V_{sw}(t) &= \varepsilon dt \\ &+e^{-\beta dt}[\delta dt \frac{1-\alpha}{2}\Delta V_{sw}(t) \\ &+(1-\delta dt)\cdot(1-2\lambda dt)\cdot \Delta V_{sw}(t+dt)]\end{aligned}$$

Expanding $\Delta V_{sw}(t+dt) = V_{sw}(t) + \frac{\partial V_{sw}(t)}{\partial t}\cdot dt$, $e^{-\beta dt} = 1-\beta dt$, and neglecting terms in dt^2:

$$\beta \Delta V_{sw}(t) = \varepsilon - (2\lambda + \frac{\delta}{2}(1+\alpha))\Delta V_{sw}(t) + \frac{\partial \Delta V_{sw}(t)}{\partial t} \tag{26}$$

We assume that the value functions V_{sw}^D and V_{sw}^R stay finite over $t \in [0,\infty)$. Hence $\Delta V_{sw}(t)$ does not diverge as $t \to \infty$. As $\beta + 2\lambda + \frac{\delta}{2}(1+\alpha) > 0$, the only bounded solution to equation 26 is a constant:

$$\Delta V_{sw}(t) = \frac{\varepsilon}{\beta + 2\lambda + \frac{\delta}{2}(1+\alpha)} \equiv e$$

This concludes the proof of the lemma. □

The lemma allows us to finish the proof of proposition 3. Going back to the Bellman equation of relaxed white households (equation 24), we use the lemma, $V_{sw}^D(t) = V_{sw}^R(t) - e$; expand $V_{sw}^R(t+dt) = V_{sw}^R(t) + \frac{\partial V_{sw}^R(t)}{\partial t}\cdot dt$, $e^{-\beta dt} = 1-\beta dt$, and neglect terms in dt^2. Then,

$$\begin{aligned}\beta V_{sw}^R(t) &= v_w - \rho b(t) \\ &+\delta\left[V_{lw} + \frac{1-\alpha}{2}\left[\left(V_{sw}^R(t) - V_{lw}\right) + (V_{sb} - V_{lb})\right] - \alpha_0 - V_{sw}^R(t)\right] \\ &+\frac{\partial V_{sw}^R(t)}{\partial t} - \lambda e\end{aligned}$$

Noting $(\frac{\delta}{2}(1+\alpha)+\beta)\mathbf{V} = \delta\left[V_{lw} + \frac{1-\alpha}{2}\left[-V_{lw} + (V_{sb} - V_{lb})\right] - \alpha_0\right] + v_w - \rho - \lambda e$, this leads to the following differential equation for the value function:

$$\frac{\partial V_{sw}^R(t)}{\partial t} - (\frac{\delta}{2}(1+\alpha)+\beta)\cdot V_{sw}^R(t) + (\frac{\delta}{2}(1+\alpha)+\beta)\mathbf{V} + \rho(1-b(t)) = 0 \tag{27}$$

And note that $\mathbf{V} = \lim_{t\to\infty} V_{sw}^R(t)$. To solve the first-order differential equation (27), define a function $\varphi(t) = V_{sw}^R(t)e^{-(\frac{\delta}{2}(1+\alpha)+\beta)t}$. Then, this function satisfies a solvable first-order equation.

$$\varphi'(t) = -((\frac{\delta}{2} + \beta)\mathbf{V} + \rho(1 - b(t))) \cdot e^{-(\frac{\delta}{2}(1+\alpha)+\beta)t} \tag{28}$$

Interestingly, $\varphi(\infty) = 0$, because the value functions of distressed and relaxed white $V_{sw}^R(t)$ and $V_{sw}^D(t)$ are assumed bounded over $t \in [0, \infty)$ and $e^{(\frac{\delta}{2}(1+\alpha)+\beta)t} \to \infty$ as $t \to \infty$. Hence the solution to equation 28 is $\varphi(t) = \int_t^\infty ((\frac{\delta}{2} + \beta)\mathbf{V} + \rho(1 - b(s))) \cdot e^{-(\frac{\delta}{2}(1+\alpha)+\beta)s} ds$. Finally, plugging back this expression of $\varphi(t)$ into the original definition of $\varphi(t) = V_{sw}^R(t)e^{-(\frac{\delta}{2}(1+\alpha)+\beta)t}$,

$$V_{sw}^R(t) = \int_t^\infty ((\frac{\delta}{2}(1 + \alpha) + \beta)\mathbf{V} + \rho(1 - b(s))) \cdot e^{-(\frac{\delta}{2}(1+\alpha)+\beta)(s-t)} ds$$

Using the dynamics of racial composition, $\frac{db}{dt} = \delta(1 - b(t))$, hence $b(t) = 1 - (1 - b(\hat{t}))e^{-\delta(t-\hat{t})}$ for $t \geq \hat{t}$. Therefore, the integral can be calculated to get the value function of the relaxed white household.

$$\begin{aligned} V_{sw}^R(t) &= \int_t^\infty ((\frac{\delta}{2}(1 + \alpha) + \beta)\mathbf{V} + \rho(1 - b(\hat{t}))e^{-\delta(s-\hat{t})}) \cdot e^{-(\frac{\delta}{2}(1+\alpha)+\beta)(s-t)} ds \\ &= \mathbf{V} + \rho(1 - b(\hat{t}))e^{-\delta(t-\hat{t})} \frac{1}{(3\frac{\delta}{2} + \alpha\frac{\delta}{2} + \beta)} \end{aligned}$$

From this expression, we can back out the tipping point $\hat{b}$. The tipping point is reached when the fraction of black households in the neighborhood $b(t)$ equals $\hat{b}$ such that:

$$\mathbf{V} + \rho \frac{1 - \hat{b}}{3\frac{\delta}{2} + \alpha\frac{\delta}{2} + \beta} - V_{lw} = V_{sb} - V_{lb} \tag{29}$$

There are then three cases in this tipping point equilibrium:

1. An interior solution $\hat{b} \in (0, 1)$ if $V_{sw}^R(\hat{t}) - V_{lw} = V_{sb} - V_{lb}$ for some $\hat{t} \in [0, \infty)$.
   Then relaxed white households sell from $t = \hat{t}$ to $t = \infty$. This is the case if there is $\hat{b} \in [0, 1]$ that satisfies inequality 29.

2. A corner solution $\hat{b} = 0$ if $V^R_{sw}(t = 0) - V_{lw} < V_{sb} - V_{lb}$.

   Then relaxed white households sell to black from $t = 0$. Note that $V^R_{sw}(t = 0) = \mathbf{V} + \rho \frac{1}{\frac{\delta}{2}(3+\alpha)+\beta}$.

3. A corner solution $\hat{b} = 1$ if $V^R_{sw}(t = \infty) - V_{lw} > V_{sb} - V_{lb}$.

   Then relaxed white households never sell to black. Note that $V^R_{sw}(t = \infty) = \mathbf{V}$.

$\square$

Lemma 2. *The value of living in the neighborhood before the tipping point, $t \le \hat{t}$, $b \le \hat{b}$, is:*

$$V_{sw}(t) = e^{A(t-\hat{t})} V_{sw}(\hat{t}) + \frac{A^{-1}}{\hat{t}-t}[1 - \exp(-A(\hat{t}-t))] \cdot \tilde{W} - \rho \int_t^{\hat{t}} e^{-A(s-t)} \cdot \begin{pmatrix} b(s) \\ b(s) \end{pmatrix} ds$$

where $V_{sw}(t) = (V^R_{sw}(t) \quad V^D_{sw}(t))'$. A and $\tilde{W}$ are defined in the body of the proof.

Proof. We start by writing the dynamic Bellman equation of relaxed and distressed whites before the tipping point. Before the tipping point, relaxed whites do not sell when matched to black; distressed whites sell.

$$\begin{aligned} V^R_{sw}(t) &= (v_w - \rho b(t))dt \\ &\quad + e^{-\beta dt}[(1 - \lambda dt) \cdot V^R_{sw}(t + dt) + \lambda dt \cdot V^D_{sw}(t + dt)] \\ V^D_{sw}(t) &= (v_w - \varepsilon - \rho b(t))dt \\ &\quad + e^{-\beta dt}[\delta dt \, [V_{lw} + (1 - \alpha)p_{Db}(t) - \alpha_0] \\ &\quad + (1 - \delta dt) \left[(1 - \lambda dt) \cdot V^D_{sw}(t + dt) + \lambda dt \cdot V^R_{sw}(t + dt)\right]] \end{aligned}$$

Using the definition of price the price $p_{Db}(t)$ provided in equation 10 of the main body of the paper:

$$V_{lw} + (1 - \alpha)p_{Db}(t) - \alpha_0 = \frac{1-\alpha}{2} V^D_{sw}(t) + \frac{1+\alpha}{2} V_{lw} + \frac{1-\alpha}{2} (V_{sb} - V_{lb}) - \alpha_0$$

Developing the derivative $V_{sw}^D(t+dt) = V_{sw}^D(t) + \frac{\partial V_{sw}^D(t)}{\partial t} \cdot dt$, the exponential $e^{-\beta dt} = 1 - \beta dt$, and ignoring terms in dt^2,

$$
\begin{aligned}
\beta V_{sw}^R(t) &= v_w - \rho b(t) + \frac{\partial V_{sw}^R(t)}{\partial t} - \lambda[V_{sw}^R(t) - V_{sw}^D(t)] \\
\beta V_{sw}^D(t) &= v_w - \varepsilon - \rho b(t) + \frac{\partial V_{sw}^D(t)}{\partial t} + \lambda[V_{sw}^R(t) - V_{sw}^D(t)] \\
&\quad + \delta\left[\frac{1-\alpha}{2} V_{sw}^D(t) + \frac{1+\alpha}{2} V_{lw} + \frac{1-\alpha}{2}[V_{sb} - V_{lb}] - \alpha_0\right]
\end{aligned}
$$

In matrix form, writing $V_{sw}(t) = \begin{pmatrix} V_{sw}^R(t) \\ V_{sw}^D(t) \end{pmatrix}$, the two previous equations can be written as a single vector-valued equation as:

$$
\begin{aligned}
\frac{\partial V_{sw}(t)}{\partial t} &= \underbrace{\begin{pmatrix} \beta + \lambda & -\lambda \\ -\lambda & \beta + \lambda - \frac{\delta}{2}(1-\alpha) \end{pmatrix}}_{A} V_{sw}(t) \\
&\quad - \underbrace{\begin{pmatrix} v_w \\ v_w - \varepsilon + \delta[\frac{1+\alpha}{2} V_{lw} + \frac{1-\alpha}{2}[V_{sb} - V_{lb}] - \alpha_0] \end{pmatrix}}_{\tilde{W}} + \rho b(t) \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}
\end{aligned}
$$

where the matrix A is a matrix of dimension 2 equal to the number of states (distressed and relaxed). The continuity of the value function at $t = \hat{t}$ is used to determine the unique closed-form solution of the valuation of the neighborhood. Since $V_{sw}(t)$ is continuous in $t = \hat{t}$, and noting $\psi(s) = -\tilde{W} + \rho b(s) \cdot (1 \quad 1)'$, the closed-form solution for white households' valuation of the neighborhood for $t \leq \hat{t}$ is, assuming the continuity of the value function.

$$\begin{aligned} V_{sw}(t) &= e^{A(t-\hat{t})} V_{sw}(\hat{t}) - \int_t^{\hat{t}} e^{-A(s-t)} \cdot \psi(s) ds \\ &= e^{A(t-\hat{t})} V_{sw}(\hat{t}) + \int_t^{\hat{t}} e^{-A(s-t)} \cdot \left[\tilde{W} - \rho b(s) \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right] ds \\ &= e^{A(t-\hat{t})} V_{sw}(\hat{t}) + \frac{A^{-1}}{\hat{t}-t} [1 - \exp(-A(\hat{t}-t))] \cdot \tilde{W} - \rho \int_t^{\hat{t}} e^{-A(s-t)} \cdot \begin{pmatrix} b(s) \\ b(s) \end{pmatrix} ds \end{aligned}$$

where $\exp(-A(\hat{t}-t))$ is the exponential of the matrix $-A(\hat{t}-t)$, defined as $\exp X = \sum_{k=0}^{\infty} \frac{1}{k!} X^k$. The value functions are decreasing with time as the fraction of blacks in the neighborhood increases. Also the value functions are linear and decreasing in ρ. □

Lemma 3. (Neighborhood composition at time t in racial transition) *The number of depressed and relaxed households, respectively $D(t)$ and $R(t)$, evolves in the following way.*

- *Before the tipping point, for $t \leq \hat{t}$, i.e. for $b \leq \hat{b}$.*

$$\begin{aligned} D(t+dt) - D(t) &= -\lambda D(t)dt + \lambda R(t)dt - \delta D(t)dt \\ R(t+dt) - R(t) &= -\lambda R(t)dt + \lambda D(t)dt \end{aligned}$$

Hence, writing $D(t+dt) - D(t) = \frac{dD}{dt} \cdot dt$, the fraction of black, distressed white, relaxed white households at time t is the solution to:

$$\frac{d}{dt} \begin{pmatrix} D(t) \\ R(t) \end{pmatrix} = \begin{pmatrix} -\lambda - \delta & \lambda \\ \lambda & -\lambda \end{pmatrix} \begin{pmatrix} D(t) \\ R(t) \end{pmatrix}$$

- *After the tipping point, for $t \geq \hat{t}$, i.e. for $b \geq \hat{b} = b(\hat{t})$,*

$$\begin{aligned} D(t+dt) - D(t) &= -\lambda D(t)dt + \lambda R(t)dt - \delta D(t)dt \\ R(t+dt) - R(t) &= -\lambda R(t)dt + \lambda D(t)dt - \delta R(t)dt \end{aligned}$$

And the total number of whites declines at rate δ, i.e. $D(t) + R(t) = e^{-\delta(t-\hat{t})}(1 - b(\hat{t}))$. Hence, after the tipping point, the fraction of black, distressed white, relaxed white households at time t is the solution to:

$$\frac{d}{dt}\begin{pmatrix} D(t) \\ R(t) \end{pmatrix} = \begin{pmatrix} -\lambda-\delta & \lambda \\ \lambda & -\lambda-\delta \end{pmatrix}\begin{pmatrix} D(t) \\ R(t) \end{pmatrix}$$

The details of the solution are provided below.

Proof. Note $C(t) = (D(t) \quad R(t))'$, $K = \begin{pmatrix} -\lambda-\delta & \lambda \\ \lambda & -\lambda-\delta \end{pmatrix}$ and $L = \begin{pmatrix} -\lambda-\delta & \lambda \\ \lambda & -\lambda \end{pmatrix}$.

Before the tipping point has been reached, note that L is a real symmetric matrix, hence diagonalizable by the spectral theorem (Courant & Hilbert 1989). There is an orthogonal matrix D such that $K = D'Diag(k_1, k_2)D$. Then the solution to the dynamics of the neighborhood is $C(t)$=$(D(t) \quad R(t))' = D'\begin{pmatrix} c_1e^{k_1t} + c_2e^{k_2t} \\ c_1e^{k_1t} + c_2e^{k_2t} \end{pmatrix}$, with c_1, c_2 chosen so that $D(0)$ equals the initial fraction of distressed whites, and $R(0) = 1 - D(0)$.

After the tipping point has been reached, note that $D(t) + R(t)$ and $D(t) - R(t)$ satisfy the following homogeneous first-order differential equations:

$$\begin{aligned} [D(t+dt) + R(t+dt)] - [D(t) + R(t)] &= -\delta\,[D(t) + R(t)] \\ [D(t+dt) - R(t+dt)] - [D(t) - R(t)] &= -(2\lambda + \delta)\,[D(t) - R(t)] \end{aligned}$$

Hence the closed form solution for $R(t) = \frac{1}{2}\{[D(t) + R(t)] - [D(t) - R(t)]\}$, and also for $D(t) = \frac{1}{2}\{[D(t) + R(t)] + [D(t) - R(t)]\}$. $\square$

Racial preferences and the Profit of Racial Transition

Proposition 4. *Profit is linear and decreasing for $\rho \geq \overline{\rho}$, where $\overline{\rho}$ is the minimum level of racial preferences for which $\hat{t} = \hat{b} = 0$.*

Proof. If racial preferences ρ are above the threshold $\overline{\rho}$, then $\hat{t} = \hat{b} = 0$. Then the p.d.v. of revenue at time $t = 0$ is $\Pi = \int_{t=0}^{t=\infty} 2\delta \left\{D(t)\kappa(p_{Db}(t)) + R(t)\kappa(p_{Rb}(t))\right\} e^{-rt}dt$. Equation (16) together with equation (17) show that the value functions $V_{sw}^{R}(t)$ and $V_{sw}^{D}(t)$ are decreasing linear functions of ρ. Hence the prices $p_{Db}(t)$ and $p_{Rb}(t)$ are linear decreasing functions of ρ and the p.d.v. of revenues Π is a linear decreasing function of ρ for $\rho \geq \overline{\rho}$. □

The Broker's Discount Rate and the Profit of Racial Transition

Proposition 5. *If the tipping point is $\hat{b} > 0$, then there are two values $\underline{r} > 0$ and $\overline{r} > 0$ of the time discount factor such that: when $r \notin [\underline{r}; \overline{r}]$, the p.d.v of revenue $\Pi = \int_0^\infty \pi(t)e^{-rt}dt$ when steering to black is smaller than the p.d.v of revenues when steering to whites π_{ss}/r.*

If the tipping point is $\hat{b} = 0$, then an impatient broker has higher revenue when steering to black than when steering to white. Formally, there is a finite $\overline{r}$ such that the p.d.v of revenues $\int_0^\infty \pi(t)e^{-rt}dt$ when steering to black is larger than the p.d.v of revenues when steering to whites for $r > \overline{r}$, and smaller if $r < \overline{r}$.

Proof. First, for $r \to 0$ (an infinitely patient broker), the sign of $\int_0^\infty \pi(t)e^{-rt}dt - \pi_{ss}/r$ is the sign of $\lim_{t\to\infty} \pi(t) - \pi_{ss}$, the difference between the revenues in an all-black neighborhood and the revenues in a steady-state white neighborhood. This is negative. Hence, there exists $\underline{r} > 0$ such that $r < \underline{r}$ implies that the p.d.v. of revenues in a neighborhood in racial transition is lower than the p.d.v of revenues in the steady-state white neighborhood.

Second, for $r \to \infty$ (an infinitely impatient broker), the sign of $\int_0^\infty \pi(t)e^{-rt}dt - \pi_{ss}/r$ is the sign of $\pi(0) - \pi_{ss}$. If the tipping point is nonzero, $\pi(0) < \pi_{ss}$, the broker makes initially a lower revenue in the neighborhood in racial transition than in the all-white neighborhood. Hence the existence of $\overline{r} > 0$ that satisfies the conditions of the proposition.

Third, if the tipping point is zero, $\pi(0) > \pi_{ss}$, and for $r \to \infty$, the p.d.v. of revenue when steering to black is higher than the p.d.v. of revenue when steering to white. Also if $\pi(0) > \pi_{ss}$, then $\pi(t)$ is a decreasing function of time, hence $\int_0^\infty \pi(t)e^{-rt}dt$ is an increasing function of r. Hence the existence of a unique $\overline{r}$ such that: the revenue of steering to black $\int_0^\infty \pi(t)e^{-rt}dt$ is larger than the steady-state all-white p.d.v. of revenues π_{ss}/r if and only if $r > \overline{r}$. $\square$

Black Households' Outside Option and the Profit of Racial Transition

Proposition 6. *Transaction prices are a decreasing function of black households' outside option. Formally,* $dp_{Db}(t)/dV_{lb} < 0$ *and* $dp_{Rb}(t)/dV_{lb} < 0$*. The tipping point is an increasing function of black households' outside option. Formally,* $d\hat{b}/dV_{sb} > 0$ *and* $d\hat{t}/dV_{sb} > 0$*. The revenue of matching white sellers to black buyers is a decreasing function of blacks' valuation of housing outside the neighborhood. There is a value* $\overline{V_{lb}}$ *such that: steering white sellers to black buyers leads to higher revenue than matching white sellers to white buyers for* $V_{lb} < \overline{V_{lb}}$*.*

Proof. First, we notice that the tipping point is an increasing function of V_{lb}. In equation 18, the valuation of the neighborhood (left-hand side of the equation) declines by $\delta(1-\alpha)/2/(\beta+(1+\alpha)\delta/2)$ when V_{lb} increases by 1, and the right hand side declines by 1 when V_{lb} increases by 1. Since $\delta(1-\alpha)/2/(\beta+(1+\alpha)\delta/2) < 1$, the tipping point is an increasing function of black households' outside option.

Second, transaction prices are a decreasing function of blacks' outside option. Indeed, $p_{Rb} = \frac{1}{2}[(V^R_{sw}(t) - V_{lw}) + (V_{sb} - V_{lb})]$ and $V^R_{sw}(t) = \mathbf{V} + e^{-\delta(t-\hat{t})}\frac{\rho(1-b(\hat{t}))}{(3+\alpha)\frac{\delta}{2}+\beta}$, with $\mathbf{V} = \Big[\delta \Big[V_{lw} + \frac{1-\alpha}{2}[-V_{lw} + (V_{sb} -$ All terms are decreasing functions of V_{lb}.

Combining the first (increasing tipping point) and the second property (decreasing prices), the broker's revenue is a decreasing function of black buyers' valuation of the outside option V_{sb}. $\square$

Extension: Two Brokers

Proposition 7. *In an equilibrium with two brokers either (i) no broker matches sellers to blacks, and both brokers match to black $\tilde{t}_1 = \tilde{t}_2 = \infty$ or (ii) one broker, say broker 1, starts matching sellers to blacks at $t = 0$, i.e. $\tilde{t}_1 = 0$. The other broker, say broker 2, matches white sellers to white buyers as long as the commission fees are larger when matching to whites, hence $\tilde{t}_2$ is the earliest point in time where $\pi_{2,w}(\tilde{t}_2) = \pi_{2,b}(\tilde{t}_2)$ and where $\pi_{2,w}(t) \geq \pi_{2,b}(t)$ for $t \leq \tilde{t}_2$.*

Proof. Solving broker 2's optimization program, we find that:

$$\begin{aligned}
\Pi_2(t) &= \pi_{2,w}(t)dt + e^{-rdt}\Pi_2(t+dt) \quad \text{for } t \leq \tilde{t}_2 \\
\Pi_{2,b}(t) &= \pi_{2,b}(t)dt + e^{-rdt}\Pi_{2,b}(t+dt) \quad \text{for } t > \tilde{t}_2
\end{aligned}$$

Hence, developing $\Pi_2(t+dt) = \Pi_2(t) + \frac{\partial \Pi_2}{\partial t}(t)dt$, $e^{-rdt} = 1 - rdt$ and neglecting terms in dt^2,

$$\begin{aligned}
r\Pi_2(t) &= \frac{\partial \Pi_2(t)}{\partial t} + \pi_{2,w}(t) \quad \text{for } t \leq \tilde{t}_2 \\
r\Pi_{2,b}(t) &= \frac{\partial \Pi_{2,b}(t)}{\partial t} + \pi_{2,b}(t) \quad \text{for } t > \tilde{t}_2
\end{aligned}$$

The second equation is valid from t to ∞, and we know that (i) no trade takes place between black sellers and any kind of buyer and that (ii) the neighborhood is entirely black for $t \to \infty$. Hence $\pi_{2,b}(\infty) = 0$ and:

$$\Pi_{2,b}(t) = \int_t^{\infty} \pi_{2,b}(s)e^{-rs}ds$$

The present discounted value of matching to blacks is equal to the present discounted value of future revenues. By definition of the equilibrium, broker 2 matches sellers to white buyers from $t = 0$ to $\tilde{t}_2$. Hence,

$$\Pi_2(t) = \Pi_{2,b}(\tilde{t}_2)e^{r(t-\tilde{t}_2)} + \int_t^{\tilde{t}_2} \pi_{2,w}(s)e^{-r(s-t)}ds$$

$\tilde{t}_2$ satisfies the following property: for $t = 0$ to $t = \tilde{t}_2$, $\Pi_2(t) \geq \Pi_{2,b}(t)$ and $\Pi_2(\tilde{t}_2) = \Pi_{2,b}(\tilde{t}_2)$. That implies:

$$\int_t^{\tilde{t}_2} \pi_{2,w}(s)e^{-r(s-t)}ds \geq \int_t^{\tilde{t}_2} \pi_{2,b}(s)e^{-r(s-t)}ds$$

for all $t \in [0, \tilde{t}_2]$. This therefore implies that $\pi_w(t) \geq \pi_b(t)$ for $t \leq \tilde{t}_2$ and $\pi_w(\tilde{t}_2) = \pi_b(\tilde{t}_2)$. $\square$

Numerical Simulations for Figures 1 to 6

Parameters were set as follows: $\beta = 6\%$, $\rho = \$1,150 \cdot \beta$, $v_w = \$9,000$, $r = 5\%$, $\varepsilon = \$120$, $V_{sb} = \$10,000$, $V_{lb} = \$7,800$, $V_{lw} = \$6,000$, $\alpha = 6\%$, $\lambda = 50\%$, $\delta = 1\%$.